Recently significant efforts have been devoted to extract the $\beta^+$ Gamow-Teller (GT) strengths in medium-mass nuclei ($A = 20$–70), particularly those related to outstanding unresolved issues in nuclear and astrophysics. The most notable issues involve those $\beta^+$ matrix elements that play roles in understanding Gamow-Teller quenching or missing strength [1–3], $\beta$ capture and nucleosynthesis in supernova processes [4], and double-$\beta$-decay processes [5]. Direct measurements of $\beta^+$ strengths have been performed with intermediate energy $(n, p)$ reactions [6–10]. However, the data are still scarce mainly because neutron beams can only be produced as a secondary beam, and so the counting rates are usually low. As a result, the energy resolution is often poor, usually 1 MeV or worse. This poor resolution can sometimes make interpretation of the data difficult [5]. Heavy-ion charge-exchange reactions such as ($^{12}$C, $^{12}$N) have also been used in measuring $\beta^+$ strengths [11, 12]. For such reactions, however, the reaction mechanisms are complicated and successive transfer reactions could dominate at energies below $E/A \approx 100$ MeV [11–13].

Isospin symmetry implies that the GT strengths in the $\beta^+$ direction can be deduced from the $\beta^-$ reactions such as $(p, n)$, using the isospin geometric factor $B(GT)_{\beta^+}/B(GT)_{\beta^-} = (T_0 + 1)(2T_0 + 1)$ for reactions on the same $N \approx Z$ target with ground state isospin $T_0$ populating analog $T_0 + 1$ final states with $T_z = T_0 \pm 1$. In contrast to $(n, p)$ reactions, resolutions of 300 keV or better can be easily achieved in $(p, n)$ reactions. Better resolution than achievable with $(n, p)$ reactions is required to make critical tests of different theoretical models, particularly in those nuclei which involve double-$\beta$ decay or supernova evolution. However, for a target with $N \geq Z$, $\beta^-$ reactions excite $T_z = T_0 - 1$ states of three different isospins: $T_0 - 1$, $T_0$, and $T_0 + 1$. The $T_0 + 1$ states are frequently obscured by states of lower isospin, especially if they are only populated weakly. In contrast, $(n, p)$ reactions on such a target populate a unique isospin $T_0 + 1$. To resolve the ambiguity about isospin, a novel idea has been proposed recently by Anantaraman et al. [14]. They argued that a comparison of $(p, n)$ and $(p, p')$ reactions with energy systematics and shell model calculations can identify the $T_0 + 1$ states unambiguously. They further demonstrated this idea for $(p, n)$ reactions on $^{26}$Mg and $^{60,62}$Ni. However, this technique, as they argued, works only for certain nuclei whose isospin is neither too small nor too large [14].

We have investigated the $^{26}$Mg($d$, $^3$He)$^{26}$Na reaction and compared it to $^{26}$Mg($p$, $n$)$^{26}$Al in order to test this idea. We conclude that, while it is indeed possible to deduce $\beta^+$ Gamow-Teller strengths from $(p, n)$ reactions, ambiguities in the large background subtraction may lead to significant uncertainties in the conclusions. $^{26}$Mg was selected for this study because both $(p, n)$ data [15] and full sd-shell-model calculations [14–16] are available; so rigorous comparisons are possible. Shell model calculations indicate that the $T_0 + 1$ strength is more sensitive to configuration mixing than is the strength of the lower isospin states. By calculating the strength for the case of $^{26}$Mg and by exploring its dependence on both the model space [full sd-shell or two-particle–two-hole (2p2h) spaces] and the coupling scheme [$jj$ or SU(3) $LS$ coupling], it was concluded that $^{26}$Mg is close to the SU(3) limit while other heavy targets, such as $^{54}$Fe, are close to the $jj$ limit [16]. Even with the full sd-shell space, 40% of additional strength for $^{26}$Mg is still missing, presumably due to 2p2h correlations outside the major shell. Because of sensitivity to details of the shell model calculations, it is important to subject them to detailed experimental tests and calibration.

The experiment was performed using 125.2 MeV deuterons from the Texas A&M University K500 superconducting cyclotron. A self-supporting $^{26}$Mg (5.3 mg/cm$^2$) target was used for the present study. An optimized detection system, the Texas A&M Proton Spectrometer [17], was used to detect the correlated protons from $^3$He decay. The Proton Spectrometer includes a magnetic spectrometer with point-to-parallel optics, two drift chambers, and X and Y scintillator trigger arrays. Each of the two drift chambers consists of five sense wire layers—two $x$ layers, one diagonal layer, and two $y$ layers. To minimize multiple scattering,
from the relation with the known Gamow-Teller strengths deduced cross sections for these nuclei show a well-defined linear $Q_E$, the excitation energy in $^{26}$Na, after correcting for the boring beam bursts. The spectra are shown as a function of backgrounds obtained from two protons triggering in neighborings $\theta \approx 7^\circ$, near the entrance to the magnet at intermediate scattering angles, and outside the magnet in the target chamber at large scattering angles ($\theta \approx 15^\circ$). Using the kinematic constraints of $(d,^2$He) reactions on several targets, notably $^1$H, $^6$Li, and $^{12}$C, we were able to determine the incident beam angle to better than 0.1° and the beam energy to better than 200 keV. The $^2$He energy resolution for the present study was 600–700 keV full width at half maximum (FWHM). The $^2$He angular resolution was better than 0.4° FWHM. We have recently measured the $(d,^2$He) cross sections on several p-shell nuclei and an sd-shell nucleus $^{24}$Mg with the Proton Spectrometer [19]. The measured 0° cross sections for these nuclei show a well-defined linear relation with the known Gamow-Teller strengths deduced from $\beta$ decay and from $(p,n)$ studies, thus providing a calibration for the present study.

Figure 1 shows the excitation functions of $^{26}$Mg$(d,^2$He)$^{26}$Na reactions measured at 0°, 3°, 6°, and 8°, respectively, after subtracting the random coincidence backgrounds obtained from two protons triggering in neighboring beam bursts. The spectra are shown as a function of $E^*$, the excitation energy in $^{26}$Na, after correcting for the reaction $Q$ value. Very few events are recorded at excitation energies $E^* < 0$ after subtracting the random backgrounds, indicating that the backgrounds were well understood. The backgrounds, however, are typically only a few counts per bin or less for Fig. 1.) To ensure that the two protons are from the $^1S_0$ state of $^2$He, an off-line cut on the relative energy of $^3$He, $E_{rel} \leq 1$ MeV, has been used. The strongest peak at $E^* \approx 0.09$ MeV, which decreases at larger angles, indicates the dominant $\Delta L=0$ transition to the first excited state of $^{26}$Na. [The transition from $^{26}$Mg(g.s.,$^3$He) to $^{26}$Na(g.s.,$^3$He) would have a different angular distribution characteristic of $\Delta L=2$.] The detailed angular distribution for this transition is shown in Fig. 2 as a function of center-of-mass angle. Overall, the measured cross sections decrease rapidly with angle, similar to $(d,^2$He) reactions on other targets we measured [19]. However, in contrast to the p-shell targets where the diffraction patterns characteristic of $\Delta L=0$ transfer are damped due to the $L=2$, tensor interactions [12,13,19], one now sees clearly a diffraction minimum near 8°. (The minimum occurs at $\theta_{c.m.} \approx 8^\circ$. The next bin at $\theta_{c.m.} \approx 8.3^\circ$, which shows the sudden increase, was obtained simultaneously. Thus data on both sides of the diffraction minimum share a common normalization.) The minimum can also be clearly seen in Fig. 1, where this state is seen to be populated strongly at $\theta_{lab} \approx 0^\circ$. It becomes comparable to the broad transitions at higher excitation energies at $\theta_{lab} \approx 3^\circ$, smaller than those at higher excitations at $\theta_{lab} \approx 6^\circ$, and become larger again at $\theta_{lab} \approx 8^\circ$.

Figure 3 shows our measured 0° $(d,^2$He) cross sections at $E_d=125.2$ MeV as a function of the corresponding Gamow-Teller strengths deduced from $\beta$-decay studies, when available, or from $(p,n)$ reactions, taken from our earlier studies [19]. Though the data are shown for nuclei in two different major shells and include transitions with momentum transfers ranging from $q \approx 0.05$ to 0.22 fm$^{-1}$ ($q$ being the half momentum transfer), a well-defined linear relation has been observed, indicating the usefulness of the $(d,^2$He) reaction to study Gamow-Teller strengths. The solid line in Fig. 3 indicates a least-squares fit of a linear relation, $d\sigma/d\Omega(0^°) = a B(GT)$, to the data from $^6$Li, $^{12}$C, $^{13}$C, and $^{24}$Mg [19]. This fit yields a slope parameter $\alpha = 1.30 \pm 0.04$.
However, is significantly smaller than the value we find to be populated strongly in $^{26}$Mg($^{3}$He) reactions at much higher energies [20]. It is also consistent with the shell-model calculation $B(\Gamma T^+)=0.48$ after inclusion of the 40% empirical correction factor [16,21]. Furthermore, it is notable that the similarities in the target masses, momentum transfers, and measured cross sections for $^{24}$Mg($^{3}$He)$^{24}$Al and $^{26}$Mg($^{3}$He)$^{26}$Al reactions imply that our deduced $B(\Gamma T)$ is essentially model independent in this case. Our new $B(\Gamma T)$ value for $^{26}$Mg, $B(\Gamma T^+)=0.44$±0.04, however, is significantly smaller than the value $B(\Gamma T^+) = 6B(\Gamma T^-) = 0.72$ deduced from the best determination, $B(\Gamma T^-) = 0.12$ for the $T=2$, $1^+$ state in $^{26}$Mg($p,n$)$^{26}$Al reactions [15]. In Figs. 3(a)–3(c) of Madey et al. [15], the narrow peak near channel 1640 was identified as a transition to the lowest $T=2$, $1^+$ state in $^{26}$Al at an excitation energy of $E_x = 13.6$ MeV. The analog of this state is the first $1^+$ excited state of $^{26}$Na at an excitation energy of 0.088 MeV that we find to be populated strongly in $^{26}$Mg($^{3}$He)$^{26}$Al. Indeed, better energy resolution, 370 keV FWHM, was achieved in the $(p,n)$ study, compared to 650 keV FWHM in our $(d,^{3}$He) reactions. However, in $(p,n)$ reactions, the $T=2$, $1^+$ states, which can be used to deduce the analog $T_0 + 1$ GT strength in the $\beta^+$ direction, are positioned on top of a large background. This background, caused mainly by $(p,\alpha)$ quasifree scattering and population of $1^+$, $T_0 - 1$, and $T_0$ final states, is comparable to or larger than the peaks of $T=2,1^+$ states. Furthermore, several smaller peaks, observed at higher excitation energies in Ref. [15], are results of overlapping states with $T=0, 1$, and 2, which are difficult to separate. In contrast, as one can see from Fig. 1, these backgrounds are absent in our $(d,^{3}$He) reaction.

In fact, three different backgrounds—(1) a calculated quasifree $(p,pn)$ background [22] plus a cosmic-ray background, (2) a polynomial background for the entire spectrum, and (3) a separate polynomial background for each of three different excitation energy regions—were considered in Ref. [15]. These different background treatments, however, yielded significantly different values in cross sections, with deviations as large as 50%, particularly in the high excitation energy region where the $T_0 + 1$ states start to be populated. It was argued [15] that, because methods (1) and (2) yielded similar results, with differences of 8% to 20% in the three regions, and because method (1) is based on a model for the observed background, it provided the best estimate of the underlying background. This led Madey et al. [15] to adopt the largest of their three sets of extracted GT strengths as their preferred value. Determining the quasifree background, however, requires the renormalization of the calculated continuum at a certain cutoff energy $E^* = 39.3$ MeV. It is not known how the extracted cross sections depend on this cutoff energy. Moreover, it is unclear whether the calculated quasifree line shape is reliable at 0° since the model has only been compared to data at angles larger than 15° [22]. Even at 15°, the calculations started to deviate from data at high excitation energies [22] where the calculated background was normalized in [15]. In contrast, Ref. [15] would have obtained a $B(\Gamma T)$ value consistent with our result if the third (largest) background had been adopted. The problem is that, given the $(p,n)$ cross section data alone, one does not known how to treat the continuum background unambiguously.

Perhaps the most important implication of this discrepancy is not for $^{26}$Mg alone. Indeed, as shown in Fig. 3, the $(p,n)$ and $(d,^{3}$He) reactions provide consistent results concerning the $\beta^+$ Gamow-Teller strengths for the other sd-shell nucleus, $^{26}$Mg, for which $T_0 = 0$ and no quasifree background is present in $(p,n)$ reactions in the region of interest [23]. As the target ground state $T_0$ increases, the $T_0 + 1$ Gamow-Teller strengths become smaller (roughly proportional to $1/T_0^2$) in $(p,n)$ reactions. At the same time, they shift to higher excitation energies in the residual, where the quasifree backgrounds become bigger. For $^{26}$Mg where $T_0 = 1$, the GT peak is already comparable to the continuum background in $(p,n)$ reactions. For this system, we find a 30% systematic uncertainty in extracting the $B(\Gamma T)$ yield from $(p,n)$ reactions. Comparable or larger uncertainties could be expected for $(p,n)$ reactions for targets with larger ground state $T_0$, such as double $\beta$-decay daughters and nuclei in the fp shell that play a role in supernova evolution. We also note that, while in $(p,n)$ reactions the GT cross sections become diminishingly smaller as $T_0$ increases due to isospin geometric factors, the corresponding $\beta^+$ GT cross sections are roughly independent of $T_0$. Thus the $T_0 + 1$ strengths can be readily measured in direct $\beta^+$ reactions.

In conclusion, we have measured $^{26}$Mg($d,^{3}$He)$^{26}$Na cross sections at 125 MeV. The Gamow-Teller strength that we determined for the $^{26}$Na first excited state is significantly smaller than that inferred in the $^{26}$Mg($p,n$)$^{26}$Al reaction, populating its analog state. Our message is simple: For targets with large ground state $T_0$ values, the $\beta^+$ Gamow-Teller
strengths for $T_{0}+1$ states deduced from $(p,n)$ and $(p, p')$
reactions, though with better resolutions, may have large sys-
tematic uncertainties due, not to detector resolutions, but to
ambiguities in the large backgrounds which must be sub-
tracted. Final conclusions concerning their GT strengths can
only be reached when a better understanding of the back-
grounds is obtained or when the detailed comparisons with
direct $\beta^{+}$ measurements, such as $(n,p)$, $(d, {^{2}}He)$, and
$(t, {^{3}}He)$, are available.

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