Nuclear matter compressibility from isoscalar giant monopole resonance

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We examine the status of the nuclear matter compressibility K_{nm} obtained from experimental data of the strength distribution of the giant monopole resonance in nuclei and employing a least-squares fit to a semiempirical expansion of the nucleus compressibility K_A in $A^{-1/3}$. We present arguments indicating that all the coefficients of this expansion must be determined by a fit to the data. In our analysis we have used the entire data set, correcting for systematic energy differences between data sets measured in different laboratories, and applying the same criteria to all sets in extracting the uncertainties. Contrary to recent statements by Sharma and collaborators, we find that the present complete data set is not adequate to limit the range of K_{nm} to better than about a factor of 1.7 (200 to 350 MeV).

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The nuclear matter ($N=Z$ and no Coulomb interaction) compressibility K_{nm} is an important quantity characterizing the nuclear medium since it is directly related to the curvature of the nuclear matter equation of state [1] $E=E(\rho)$, at the saturation point $(E,\rho)=(-16$ MeV, 0.17 fm^3). An accurate determination of K_{nm} is very important for the study of properties of nuclei (radii, masses, giant resonances, etc.), supernova collapses, neutron stars, and heavy-ion collisions [2].

The study of the isoscalar giant monopole resonances (GMR) in various nuclei provides an important source of information for K_{nm}. The GMR was first discovered in ^{208}Pb at an excitation energy [3] of 13.7 MeV. Random-phase approximation (RPA) calculations using existing or modified effective interactions having $K_{nm}=210\pm30$ MeV were in agreement with experiment [4]. It is important to note, however, that this commonly accepted value of $K_{nm}=210\pm30$ MeV was deduced using a limited class of effective interactions.

With the increase in GMR data in various nuclei, it became worthwhile to attempt using a semiempirical approach to deduce K_{nm}. In this approach, which is similar to the semiempirical mass formula, one writes [4,5] the compressibility K_A of the nucleus with mass number A as an expansion in $A^{-1/3}$,

$$K_A=K_{vol}+K_{surf}A^{-1/3}+K_{curv}A^{-2/3}+(K_{sym}+K_{ss}A^{-1/3})\left[\frac{N-Z}{A}\right]^2+K_{Coul}\frac{Z^2}{A^{4/3}}+\cdots , \quad (1)$$

where K_A is defined by

$$K_A=\frac{m}{\hbar^2}E_{GMR}^2\langle r^2\rangle . \quad (2)$$

Here $\langle r^2\rangle$ is the mean-square radius of the nucleus, and E_{GMR} is taken to be the scaling energy of the GMR, defined by

$$E_{GMR}=\sqrt{m_3/m_1} , \quad (3)$$

where m_k is the RPA sum rule

$$m_k=\sum_n (E_n-E_0)^k|\langle 0|r^2|n\rangle|^2 . \quad (4)$$

Note that with the definition (3) for E_{GMR}, K_{vol} in (1) is equal [5] to K_{nm}.

There have been several attempts in the past [5–12] to determine K_{nm} using the procedure described by (1)–(4) by a least-squares (LS) fit to the GMR data of various sets of nuclei. In these attempts, only a very limited number of parameters (1–3), mainly K_{vol}, K_{surf}, and K_{sym}, were included in the LS fit. Fixed values (deduced from theory) were used for the other parameters, such as K_{Coul} and K_{curv}, in Eq. (1). Recently, Sharma and collaborators in a series of papers [10–12] claimed that a value of $K_{nm}=300\pm20$ MeV is obtained using the recent GMR data of Groningen for the nuclei ^{112,114,116,120,124}Sn and ^{144,148}Sm, and including those of ^{208}Pb, ^{90}Zr, and ^{24}Mg. It should be pointed out that this result is quite different from the commonly accepted value of $K_{nm}=210\pm30$ MeV. Very recently, Pearson [13] has pointed out that K_{nm} is strongly dependent on the value assumed for K_{Coul} and that the relation between K_{Coul} and K_{nm} is model dependent.

In the present study, we take a closer look at the semiempirical analysis of the GMR data, using the procedures (1)–(4) in an attempt to extract a reliable value for K_{nm}. We find that the claim of Sharma *et al.* [10–12] is not reliable since their analysis is limited in the number of data points and the number of free parameters included in the LS fit. We have attempted to include the entire GMR data set, reconciling differences between different laboratories and taking the parameters in Eq. (1) as free parameters. In the following, we first summarize some theoretical and experimental observations concerning the procedure of Eqs. (1)–(4) and then provide some numerical results and conclusions.

We now discuss the following considerations that must be taken into account when using Eqs. (1)–(4) in a fit to the experimental data for the GMR.

(1) In using (3) to determine E_{GMR}, the entire GMR

energy-weighted sum rule (EWSR) must be known experimentally. This appears to be the case for heavy nuclei where the GMR strength is fitted by a Gaussian with centroid E_0 and width Γ. In this case one has

$$E_{\rm GMR}^2 = E_0^2 + 3(\Gamma/2.35)^2 \ . \tag{5}$$

(2) In deformed nuclei, a splitting of the GMR strength into clearly identifiable components occurs [14–17]. In this case, Eq. (3) cannot be used to obtain $E_{\rm GMR}$, which corresponds to the spherical configuration. Theoretical considerations indicate [17] that to a good approximation the higher component is shifted upward by an amount proportional to the deformation parameter β. We have therefore included in our analysis the GMR data for deformed nuclei by adopting the centroid E_0 with width Γ of the higher component and adding to Eq. (1) the term

$$\beta K_{\rm def} \ . \tag{6}$$

(3) At present, any attempt to include GMR data for light nuclei should be considered with extreme care due to the following reasons: (i) RPA calculations of the GMR predict that the strength is fragmented [18,19] over quite a large range (over 10 MeV). Therefore, GMR strength must be carefully searched for over a wide range of energy. (ii) The particle decay width [18,20] of the GMR is quite large (5–10 MeV), particularly for high-energy components. This makes the experimental task of determining the GMR strength distribution rather difficult. (iii) For light nuclei, the scaling approximation may not be as good an approximation as in the case of heavy nuclei, introducing [19] errors of about 5% in the determination of $E_{\rm GMR}$ from Eq. (3). In this work we also discuss the implication of the present data on GMR in light nuclei.

(4) In determining K_A from Eq. (2), one usually adopts a certain expression for $\langle r^2 \rangle$ with a specific $A^{1/3}$ dependence. The $A^{1/3}$ dependence of $\langle r^2 \rangle$ affects the $A^{1/3}$ expansion of K_A. Since different expressions for $\langle r^2 \rangle$ will lead to different values for the coefficients in the expansion (1) for K_A, adopting theoretical values for some of the coefficients will be inconsistent.

(5) In previous analyses of the GMR data, such as in Refs. [5, 10–12], the number of free parameters in (1) was reduced by adopting relations between the parameters, such as $K_{\rm surf} = -K_{\rm vol}$ and

$$K_{\rm Coul} = \frac{3e^2}{5r_0}\left[\frac{1215}{K_{\rm nm}} - 12.5\right] \ {\rm MeV} \ , \tag{7}$$

obtained from theory [5]. It should be pointed out that these relations were derived using a limited class of effective interactions, and they are not unique [13]. Therefore, from points (4) and (5) we conclude that all parameters of Eq. (1) should be determined by a least-squares fit to the experimental GMR data.

Extensive investigations of the giant monopole resonance have occurred at three laboratories: Texas A&M, Grenoble, and Groningen. Each has taken spectra into the very small angles necessary to separate monopole from quadrupole strength, a technique pioneered at Texas A&M (TAMU) [3]. At Texas A&M, substantial monopole strength was identified in 17 nuclei using inelastic α scattering between 96 and 130 MeV [3,8,21–24]. At Grenoble, monopole strength was observed in 42 nuclei with 100-MeV ^{3}He scattering [7] and in three nuclei with α scattering [25]. At Groningen 13 nuclei were investigated with 120 MeV α scattering [10,26–29]. The nuclei investigated and the resulting monopole parameters are summarized in Table I.

It is immediately apparent that the ^{3}He data yields a much lower monopole strength and somewhat smaller widths than the α data for $A \leq 154$. The Grenoble group later investigated three nuclei with α scattering and obtained results in good agreement with the other α experiments. They conclude [25] that the GMR structure "seems to extend further up the high excitation-energy side" in α scattering and provide the possibility that this difference "can be due in part to the choice of subtracted background." In any case, only a portion of the GMR strength is seen in the ^{3}He scattering.

Of these 75 potential data points, only 27 (9 TAMU, 11 Groningen, 2 Grenoble α, 5 Grenoble ^{3}He) have EWSR fractions consistent with 100% of the monopole strength. These 27 data points represent 16 different nuclei with $24 \leq A \leq 232$.

At Groningen, a special effort was made to measure GMR parameters in the Sn and Sm isotopes precisely [10]. Spectra were taken over the range 0–3°, and ray tracing was used to divide the results into two spectra: one 0–1.5° where the monopole is strong, and one 1.5–3° where the monopole is weak. The larger-angle spectrum was then subtracted from the first to enhance the monopole, and the resulting spectrum was fit to determine monopole parameters. They reported substantially smaller errors in position of the monopole than other works, though their errors in width are comparable to others.

Errors in GMR parameters from each of these works have three components: (1) Uncertainties in the fitting process due to statistical errors and the appropriateness of the model, usually obtained from the error matrix. (2) Uncertainties in subtraction of the continuum and background, which are subjective and probably arrived at differently by the different groups. (3) Systematic uncertainties such as energy calibration, absolute yield calibration, etc.

If all of the available data are to be used, it is important to attempt to put the data from different laboratories on a similar footing. Both the actual value of the energy and width and their uncertainties are important. Systematic differences between different data sets will distort the fits, and data with lower stated uncertainties will dominate the fits. Thus we have explored both the parameters and the uncertainties reported by the different laboratories.

We looked for energy-calibration systematic differences by comparing energies obtained for both the GMR and the nearby giant quadrupole resonance (GQR) by the three laboratories. There are eight nuclei measured by both TAMU and Groningen, and comparisons of GQR and GMR energies are shown in Table II. Of the 16

comparisons, in 13 cases Groningen's energies are higher than those obtained at TAMU. The 16-point average shows Groningen energies 290 keV higher than the TAMU energies. Two of the three nuclei measured with α scattering at Grenoble were also measured at TAMU. For all four data points, the Grenoble energies were higher, the average being 220 keV. Only the GQR energies were used in the ^{3}He comparison.

TABLE I. Giant monopole resonance parameters.

A	Nucleus	E_x	Γ	% EWSR	Ref.[a]
28	Si[b]	19.00±0.50	6.30±0.50	66±12	TAMU1
58	Ni	17.00±0.40	4.00±0.40	19±10	TAMU2
64	Zn	18.20±0.50	4.30±0.90	29±16	TAMU3
66	Zn	18.40±0.70	4.10±1.10	30±16	TAMU3
90	Zr	16.20±0.50	3.50±0.30	90±20	TAMU3
112	Sn	15.70±0.30	4.20±0.30	79±25	TAMU4
116	Sn	15.60±0.30	4.10±0.30	180±60	TAMU3
118	Sn	15.50±0.60	4.10±0.70	150	TAMU3
120	Sn	15.20±0.50	4.10±0.60	180	TAMU3
124	Sn	14.80±0.40	3.80±0.60	186±60	TAMU3
142	Nd	14.80±0.30	3.30±0.20	80±20	TAMU5
146	Nd	15.10±0.30	3.50±0.20		TAMU5
150	Nd	15.40±0.30	3.40±0.20	50±15	TAMU5
144	Sm	14.60±0.20	3.00±0.30	140±40	TAMU3
148	Sm	14.60±0.20	2.80±0.30	95±25	TAMU6
154	Sm	14.90±0.30	2.60±0.40	55±15	TAMU3
208	Pb	13.70±0.40	3.00±0.50	90±20	TAMU3
		Groningen α, α			
24	Mg	17.20		90±20	Gron1
40	Ca	14.40		30±6	Gron2
90	Zr	16.10±0.28	3.10±0.28		Gron3
112	Sn	15.80±0.14	3.30±0.25	106±24	Gron4
114	Sn	15.80±0.14	3.50±0.29	107±23	Gron4
116	Sn	15.60±0.16	3.70±0.39	101±22	Gron4
120	Sn	15.50±0.15	3.90±0.35	94±20	Gron4
124	Sn	15.30±0.16	3.40±0.35	108±22	Gron4
144	Sm	15.10±0.14	3.30±0.21	125±26	Gron4
148	Sm	14.90±0.14	3.20±0.29	117±27	Gron4
150	Sm	14.90±0.18	2.80±0.50	99±35	Gron4
152	Sm	15.50±0.18	3.10±0.52	86±25	Gron4
208	Pb	13.90±0.30	2.40±0.30		Gron5
		Julich α, α			
208	Pb	13.80±0.30	2.60±0.30	85	Juli
232	Th	13.80±0.40	3.00±0.50	66±13	Juli
238	U	13.70±0.40	3.00±0.50	65±13	Juli
		Grenoble α, α			
58	Ni	17.30±0.20	3.10±0.20	23±5	Gren1
92	Mo	16.20±0.20	4.80±0.30	85±17	Gren1
120	Sn	15.40±0.40	4.00±0.30	110±22	Gren1
		Grenoble ^{3}He, ^{3}He			
27	Al	17.10±0.20	2.50±0.40	6	Gren2
45	Sc	16.10±0.50	3.20±0.70	10	Gren2
55	Mn	17.70±0.50	4.00±0.50	10	Gren2
56	Fe	17.00±0.30	3.60±0.30	10	Gren2
58	Ni	17.10±0.30	2.50±0.30	10	Gren2
60	Ni	17.00±0.30	2.70±0.30	8	Gren2
66	Zn	17.90±0.25	2.40±0.25	16	Gren2
68	Zn	17.80±0.25	3.00±0.30	18	Gren2
89	Y	16.30±0.25	3.10±0.25	47	Gren2
90	Zr	16.40±0.25	3.60±0.25	39	Gren2
96	Zr	15.80±0.30	3.20±0.30	28	Gren2
92	Mo	16.30±0.30	4.00±0.30	24	Gren2
96	Mo	16.40±0.30	3.50±0.30	19	Gren2
100	Mo	16.00±0.45	3.70±0.45	17	Gren2
107	Ag	15.90±0.30	3.70±0.35	26	Gren2
108	Pd	16.20±0.30	4.00±0.30	29	Gren2
110	Pd	15.70±0.30	3.70±0.35	42	Gren2
110	Cd	15.90±0.25	4.10±0.25	36	Gren2
112	Cd	15.70±0.25	3.60±0.35	43	Gren2
114	Cd	15.40±0.25	4.00±0.40	53	Gren2
116	Cd	15.70±0.25	3.40±0.30	53	Gren2
115	In	15.90±0.30	2.70±0.30	27	Gren2
112	Sn	16.10±0.25	3.00±0.25	42	Gren2
116	Sn	15.50±0.25	3.20±0.30	45	Gren2
120	Sn	15.40±0.25	3.20±0.30	50	Gren2
124	Sn	14.80±0.25	3.20±0.30	58	Gren2
139	La	15.00±0.25	2.70±0.25	59	Gren2
140	Ce	14.80±0.20	3.00±0.20	57	Gren3
141	Pr	14.90±0.25	2.60±0.25	63	Gren2
144	Sm	14.70±0.20	2.90±0.20	64	Gren3
150	Sm	15.10±0.25	3.00±0.25	49	Gren3
152	Sm	14.80±0.25	3.10±0.25	42	Gren3
154	Sm	15.00±0.30	3.30±0.30	44	Gren2
159	Tb	14.80±0.25	3.40±0.25	42	Gren3
165	Ho	15.00±0.25	2.70±0.30	41	Gren3
169	Tm	14.70±0.30	2.50±0.30	40	Gren3
175	Lu	14.40±0.30	3.00±0.30	55	Gren3
181	Ta	14.20±0.25	2.50±0.25	59	Gren3
197	Au	13.40±0.20	2.40±0.20	99	Gren3
208	Pb	13.20±0.30	2.80±0.25	92	Gren3
209	Bi	13.30±0.30	2.30±0.30	108	Gren2
232	Th	13.30±0.40	2.30±0.40	84	Gren3

[a]

	Ref.		Ref.		Ref.
Gron1	[26]	TAMU1	[8]	Gren1	[25]
Gron2	[27]	TAMU2	[23]	Gren2	[7]
Gron3	[28]	TAMU3	[6]	Gren3	[14]
Gron4	[10]	TAMU4	[22]	Juli	[30]
Gron5	[29]	TAMU5	[21]		
		TAMU6	[24]		

[b] Actual centroid of strength reported in Ref. [8] (see text).

In comparing the uncertainties, we note that the Groningen group reports [10] statistical uncertainties in peak position of typically 70–90 keV. In the TAMU work, these were in the same range, suggesting that the statistical accuracy of the data from the two laboratories is comparable. The statistical contribution to the uncertainties was not reported for the Grenoble work. Systematic differences can be reduced by shifting one of the data sets by the average difference in Table II.

This leaves the subjective uncertainties due to subtraction of the continuum and background as the difference between the results from TAMU and Groningen. The primary advantage of the Groningen measurement is that spectra with strong and weak monopole contributions were taken simultaneously, so the experimental conditions do not change between the sets of data. This does lead to excellent subtraction of the quadrupole resonance for which the yield over this angular range is almost constant. For the continuum and background, however, the situation is not as good. The continuum and any background (slit scattering, etc.) are often angle dependent and, for this reason, the subtraction technique may not help. As the Groningen data is limited to the excitation range $10 \leq E_x \leq 20$ MeV, the spectra give few clues as to the shape of the background above or below the giant resonances. In fact the continuum is not apparent in the spectra shown [10], and there is little basis for determining a continuum shape. Thus, uncertainties due to continuum subtraction could be quite large. The TAMU data extend over a much wider range of energy ($5 \leq E_x \leq 80$ MeV), allowing a better determination of the continuum shape, and for most nuclei several runs were taken at each angle in differing sequences to reduce errors due to changing experimental conditions. The uncertainties added for continuum subtraction were conservative. Typically, they were chosen as the most a peak energy could be changed by differing continuum assumptions that could not be totally ruled out, ranging from one that changed slowly under the peaks (similar to the background chosen in the Groningen work [10]) to one that began near the peaks and increased rapidly, joining to a smooth extrapolation of the continuum above the resonance but excluding ^{5}He and ^{5}Li breakup. The Groningen data cannot distinguish these different background possibilities. Thus it is not clear that the overall uncertainties in peak position in the Groningen work are smaller than in the TAMU work.

We have not considered data taken only at larger angles where the GMR cannot be unambiguously identified except in the case of Th and U, where essentially no other data exists. Morsch *et al.* [30] measured ^{208}Pb, ^{232}Th, and ^{238}U using 172-MeV α particles, and we have included this data because these are the heaviest nuclei in which the GMR has been seen. As the ^{208}Pb GMR position agrees with the TAMU value, no correction was made to the Julich work for systematic differences.

The silicon data point requires a special comment. The GMR centroid given [8], as 17.9 MeV for ^{28}Si is incorrect. The actual centroid and width for the strength reported in the paper are 19.0 MeV and 6.3 MeV. The correct values are listed in the tables.

All of the points available for fitting are shown in Table III, where both the original measured parameters and those corrected for systematic differences with the TAMU work are shown. We chose to accept the TAMU energies and modify the other works by the average difference to correct for systematic errors. At present there is little experimental reason to pick one of the data sets as more accurate on an absolute basis. Where multiple measurements of the same parameter are available, weighted averages (done after correction for systematic errors) are also given.

In Table IV we present results of a least-squares fit of selected data sets of GMR to the semiempirical procedure described by Eqs. (1)–(6). Using the seven data

TABLE II. Comparison of GQR and GMR energies obtained. All units are in MeV.

	GQR		GMR		TAMU−GRON	
A	TAMU	GRON	TAMU	GRON	GQR	GMR
(a) Groningen vs TAMU α						
90	14.0	14.00	16.2	16.10	0	0.10
112	13.3	13.51	15.7	15.88	−0.21	−0.18
116	13.2	13.39	15.6	15.69	−0.17	−0.09
120	12.7	13.24	15.2	15.52	−0.54	−0.32
124	12.3	13.02	14.8	15.35	−0.68	−0.55
144	12.2	12.70	14.6	15.13	−0.50	−0.53
148	12.1	12.66	14.6	14.85	−0.56	−0.25
208	11.0	10.90	13.7	13.90	0.10	−0.2
			Average difference		−0.32	−0.25
			Standard deviation		0.27	0.20
(b) Grenoble ^{3}He vs TAMU ($A < 140$)						
40	17.70	18.20			−0.50	
48	16.20	17.10			−0.90	
58	16.07	16.20	17.01	17.1	−0.13	−0.09
60	16.31	15.90			.41	
66	14.90	14.80	18.4	17.9	.10	0.50
90	14.00	14.05	16.2	16.4	−0.05	−0.20
112	13.30	13.65	15.7	16.1	−0.35	−0.40
116	13.2	13.15	15.6	15.55	0.05	0.05
120	12.7	12.75	15.2	15.45	−0.05	−0.25
124	12.3	12.35	14.8	14.85	−0.05	−0.05
			Average difference		−0.15	
			Standard deviation		0.34	
(c) Grenoble ^{3}He vs TAMU ($A > 140$)						
144	12.2	12.25	14.6	14.7	−0.05	−0.10
154	11.8	11.70	14.9	15.0	0.10	−0.10
208	11.0	10.60	13.7	13.2	0.40	0.50
			Average difference		0.15	
			Standard deviation		0.19	
(d) Grenoble α vs TAMU						
58	16.07	16.39	17.01	17.31	−0.32	−0.30
120	12.70	12.75	15.20	15.42	−0.05	−0.22
			Average difference		−0.18	−0.26
			Standard deviation		0.11	0.11

TABLE III. GMR parameters used for compressibility fits.

		Corrected[a] for systematic difference		Values adopted for calculations				
A	Nucleus	E_x (MeV)	$\sigma(E_x)$ (MeV)	E_x (MeV)	$\sigma(E_x)$ (MeV)	Γ (MeV)	$\sigma(\Gamma)$ (MeV)	Ref.[c]
24	Mg	16.71	0.23	16.71		4.73		Gron1
28	Si[b]	19.06	0.50	19.06	0.50	6.30	0.50	TAMU1
40	Ca	14.11	0.23	14.11	0.23			Gron2
58	Ni	17.00	0.40					TAMU2
58	Ni	17.08	0.23	17.06	0.20	3.28	0.18	Gren1
64	Zn	18.20	0.50	18.20	0.50	4.30	0.90	TAMU3
66	Zn	18.40	0.70	18.40	0.70	4.10	1.10	TAMU3
90	Zr	16.20	0.50					TAMU3
90	Zr	15.81	0.36	15.95	0.29	3.29	0.20	Gron3
92	Mo	15.98	0.23	15.98	0.23	4.80	0.30	Gren1
112	Sn	15.70	0.30					TAMU4
112	Sn	15.59	0.27	15.64	0.20	3.67	0.19	Gron4
114	Sn	15.51	0.27	15.51	0.27	3.52	0.29	Gron4
116	Sn	15.60	0.30					TAMU3
116	Sn	15.40	0.28	15.50	0.20	3.96	0.24	Gron4
118	Sn	15.50	0.60	15.50	0.60	4.10	0.70	TAMU3
120	Sn	15.20	0.50					TAMU3
120	Sn	15.23	0.27					Gron4
120	Sn	15.18	0.41	15.21	0.21	3.98	0.21	Gren1
124	Sn	14.80	0.40					TAMU3
124	Sn	15.06	0.28	14.98	0.23	3.50	0.30	Gron4
142	Nd	14.80	0.30	14.80	0.30	3.30	0.20	TAMU5
144	Sm	14.60	0.20					TAMU3
144	Sm	14.84	0.27	14.69	0.16	3.23	0.17	Gron4
146	Nd	15.10	0.20	15.10	0.30	3.30	0.30	TAMU5
148	Sm	14.60	0.20					TAMU6
148	Sm	14.66	0.27	14.62	0.16	3.08	0.23	Gron4
150	Nd	15.40	0.30	15.40	0.30	3.40	0.20	TAMU5
150	Sm	14.68	0.29	14.68	0.29	2.86	0.50	Gron4
152	Sm	15.27	0.29	15.27	0.29	3.13	0.52	Gron4
154	Sm	14.90	0.30	14.90	0.30	2.60	0.40	TAMU3
208	Pb	13.70	0.40					TAMU3
208	Pb	13.63	0.38					Gron5
208	Pb	13.80	0.30	13.73	0.20	2.58	0.20	Juli
232	Th	13.80	0.40	13.80	0.40	3.00	0.50	Juli
238	U	13.70	0.40	13.70	0.40	3.00	0.50	Juli
27	Al	17.00	0.39	17.00	0.39	2.50	0.40	Gren2
45	Sc	15.95	0.60	15.95	0.60	3.25	0.70	Gren2
55	Mn	17.55	0.60	17.55	0.60	4.00	0.50	Gren2
56	Fe	16.85	0.45	16.85	0.45	3.60	0.30	Gren2
58	Ni	16.95	0.45	16.95	0.45	2.50	0.30	Gren2
60	Ni	16.85	0.45	16.85	0.45	2.70	0.30	Gren2
66	Zn	17.75	0.42	17.75	0.42	2.40	0.25	Gren2
68	Zn	17.65	0.42	17.65	0.42	3.00	0.30	Gren2
89	Y	16.20	0.42	16.20	0.42	3.10	0.25	Gren2
90	Zr	16.25	0.42	16.25	0.42	3.60	0.25	Gren2
96	Zr	15.65	0.45	15.65	0.45	3.20	0.30	Gren2
92	Mo	16.20	0.45	16.20	0.45	4.00	0.30	Gren2
96	Mo	16.25	0.45	16.25	0.45	3.50	0.30	Gren2
100	Mo	15.85	0.56	15.85	0.56	3.75	0.45	Gren2
107	Ag	15.75	0.45	15.75	0.45	3.70	0.35	Gren2
108	Pd	16.10	0.45	16.10	0.45	4.00	0.30	Gren2
110	Pd	15.55	0.45	15.55	0.45	3.75	0.35	Gren2
110	Cd	15.80	0.42	15.80	0.42	4.15	0.25	Gren2
112	Cd	15.60	0.42	15.60	0.42	3.60	0.35	Gren2
114	Cd	15.30	0.42	15.30	0.42	4.00	0.40	Gren2

TABLE III. (*Continued*).

		Corrected[a] for systematic difference		Values adopted for calculations				
A	Nucleus	E_x (MeV)	$\sigma(E_x)$ (MeV)	E_x (MeV)	$\sigma(E_x)$ (MeV)	Γ (MeV)	$\sigma(\Gamma)$ (MeV)	Ref.[c]
116	Cd	15.60	0.42	15.60	0.42	3.40	0.30	Gren2
115	In	15.75	0.45	15.75	0.45	2.70	0.30	Gren2
112	Sn	15.95	0.42	15.95	0.42	3.00	0.25	Gren2
116	Sn	15.40	0.42	15.40	0.42	3.20	0.30	Gren2
120	Sn	15.30	0.42	15.30	0.42	3.25	0.30	Gren2
124	Sn	14.70	0.42	14.70	0.42	3.20	0.30	Gren2
139	La	14.85	0.42	14.85	0.42	2.70	0.25	Gren2
140	Ce	14.95	0.28	14.95	0.28	3.00	0.20	Gren3
141	Pr	15.05	0.31	15.05	0.31	2.60	0.25	Gren2
144	Sm	14.85	0.28	14.85	0.28	2.90	0.20	Gren3
150	Sm	15.25	0.31	15.25	0.31	3.00	0.25	Gren3
152	Sm	14.95	0.31	14.95	0.31	3.10	0.25	Gren3
154	Sm	15.15	0.36	15.15	0.36	3.30	0.30	Gren2
159	Tb	15.00	0.31	15.00	0.31	3.40	0.25	Gren3
165	Ho	15.15	0.31	15.15	0.31	2.70	0.30	Gren3
169	Tm	14.85	0.36	14.85	0.36	2.50	0.30	Gren3
175	Lu	14.55	0.36	14.55	0.36	3.00	0.30	Gren3
181	Ta	14.35	0.31	14.35	0.31	2.50	0.25	Gren3
197	Au	13.60	0.28	13.60	0.28	2.40	0.20	Gren3
208	Pb	13.35	0.36	13.35	0.36	2.80	0.25	Gren3
209	Bi	13.45	0.36	13.45	0.36	2.30	0.30	Gren2
232	Th	13.50	0.44	13.50	0.44	2.30	0.40	Gren3

[a]Excitation energies are shifted by Δ for the data by the amount indicated below (from Table II). Standard deviations are $[\sigma(E_x)^2+\sigma(\Delta)^2]^{1/2}$.

	Δ (MeV)	$\sigma(\Delta)$ (MeV)
Groningen α	0.29	0.23
Julich α	0	0
Grenoble α	0.22	0.11
Grenoble ^{3}He $A<140$	0.15	0.34
Grenoble ^{3}He $A>139$	−0.15	0.19

[b]Actual centroid of strength reported in Ref. [8] (see text).
[c]See Table I.

points of Sn and Sm isotopes considered by Sharma *et al.* [10–12], we obtained similar results when we took only $K_{\rm vol}$, $K_{\rm surf}$, and $K_{\rm sym}$ as free parameters with $K_{\rm Coul}=-6.20$ MeV, deduced from (7) using $K_{\rm nm}=300$ MeV. There is a strong correlation between the values of $K_{\rm Coul}$ and $K_{\rm nm}$ as demonstrated by Fig. 1. It is seen from Table IV that the uncertainties in the parameters increase dramatically when $K_{\rm Coul}$ is added as a free parameter. This is easily understood by noticing that for the nuclei considered, the coefficient of $K_{\rm Coul}$ in (1) increases approximately as $A^{2/3}$. Therefore, varying $K_{\rm Coul}$, by changing its sign, for example, has a large effect on the value extracted for $K_{\rm vol}$. This result is in agreement with the observation by Pearson [13]. The sensitivity of the value of $K_{\rm nm}$ to the number of parameters included in the fit is easily understood if one takes into consideration that the variation in $A^{1/3}$ is only $\pm 8\%$ and the uncertainty in K_A is about 2% (the accuracy adopted by Sharma *et al.* for the $E_{\rm GMR}$ is about 1%). Including the data points of ^{208}Pb, ^{90}Zr, and ^{24}Mg adopted by Sharma *et al.* helps to reduce the uncertainties in the parameters, but with the inclusion of the parameter $K_{\rm curv}$ in the fit it is seen that the claim of Sharma *et al.* that $K_{\rm nm}=300\pm 30$ MeV is unjustifiable (see also Ref. [31]).

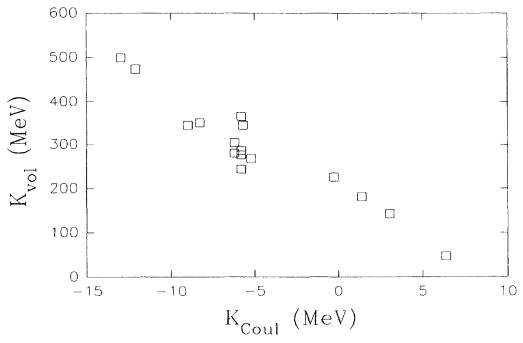

FIG. 1. Correlation between $K_{\rm vol}$ and $K_{\rm Coul}$. The results shown were obtained using the various GMR data sets and parameter sets given in Table IV.

TABLE IV. Results from fitting Eq. (1) to various data sets. Fixed parameters are indicated by their values without uncertainties. The reduced χ^2 given was obtained by dividing the sum of the squares by the number of data points minus the number of parameters.

Data	No. data points	No. of parameters	K_{vol}	K_{surf}	K_{curv}	K_{sym}	K_{Coul}	K_{def}	χ^2
Sharma	7	3	304±37	−671±177	0	−367±156	−6.20		0.023
Sn+Sm		4	344±756	−798±2379	0	−464±1835	−9.02±53		0.029
Sharma	10	3	280±9	−564±29	0	−284±136	−6.20		0.28
add Pb,Zr,Mg		4	226±48	−441±110	0	−162±173	−0.26±5.2		0.11
		5	474±823	−1750±4331	1915±6336	−566±1350	−12.11±40		0.11
Table III	19	4	286±29	−627±131	0	−264±179	−5.80	35±13	0.40
α		5	182±177	−296±567	0	−27±434	1.41±12	34±13	0.41
$A > 90$		5	365±147	−1431±1468	2032±3694	−274±180	−5.80	34±13	0.41
		5	47±1236	563±7822	−1597±14446	139±1617	6.38±47	35	0.41
Table III	41	4	276±22	−577±97	0	−261±147	−5.80	34±9	0.28
$\alpha+{}^3$He		5	268±154	−549±493	0	−242±368	−5.20±11	34±9	0.29
$A \geq 89$		5	286±128	−673±1277	243±3212	−263±149	−5.80	34±9	0.29
		5	350±1082	−1075±6823	971±12552	−351±1429	−8.28±41	35	0.29
Table III	46	4	244±16	−427±65	0	−142±133	−5.80	38±8	0.40
$\alpha+{}^3$He		5	143±53	−145±155	0	34±159	3.04±4	36±8	0.31
$A \geq 89$		5	344±49	−1270±393	1785±821	−280±148	−5.80	37±8	0.29
add Zn, ^{40}Ca[a], and ^{28}Si[a]		5	499±404	−2122±2224	3109±3481	−514±642	−12.97±19	35	0.29

[a]See text for data for ^{40}Ca and ^{28}Si.

Using the data of the GMR shown in Table III, we have performed fits for several data sets as shown in Table IV. Using all the α scattering data for $A \geq 90$ (19 points) we obtained similar results to those obtained with the seven data points adopted by Sharma *et al.* It should be emphasized that we assumed uncertainties in E_{GMR} about twice those adopted by Sharma *et al.* We then explored including additional data points where the entire sum rule is not seen and cannot be accounted for. First we added all ^{3}He data points with $A \geq 89$. This reduced the uncertainties slightly. Then we added the lighter nuclei (Zn, Ca, Si), where not all the strength was seen, to ascertain the effects on the fits. For Ca we arbitrarily assumed $E_x \approx 18 \pm 1$ MeV, as there is some evidence [7,8] of monopole strength coincident with the GQR. For ^{28}Si we assumed $E_x \approx 20 \pm 1$ MeV because only 65% of the strength was observed with a centroid of 19 MeV. Probably the rest of the strength lies higher. In this fit we left out the ^{24}Mg point, since it is much lower than for ^{28}Si. This resulted in substantially smaller uncertainties for the parameters and illustrates the importance of including lighter nuclei. The parameters themselves observed in the last case are only illustrative, however, due to the speculative nature of the Ca and Si energies.

From the results in Table IV, it is clear that K_{Coul} has a large effect on K_{nm}, and including it as a free parameter leads to uncertainties of approximately 50%, except for the ten-point Groningen data set. For this data set, with K_{Coul} as a free parameter $K_{nm} \approx 226$ MeV, rather than the ≈ 300 MeV found by Sharma. Including both K_{Coul} and K_{curv} as parameters leads to errors exceeding 100% for all coefficients. On the other hand, K_{def} is well defined at about 35 MeV, and fixing it at this value or allowing it to vary has little effect. Adding the ^{3}He data points with $A \geq 89$ reduces the uncertainties in the parameters slightly. Finally, including the data for elements lighter than Zr helps to further reduce the uncertainties in the parameters. However, as seen from Table IV, it is not possible to pin down the value of K_{nm} with an accuracy of better than 50%.

Thus, the present complete data set is clearly not adequate to limit the range of K_{nm} to better than about a factor of 1.7 (200–350 MeV). Several things need to be done to pin down K_{nm}. We need measurements on considerably more than 16 nuclei and with more variation in mass. To the extent possible, spherical nuclei should be chosen to eliminate effects of deformation. These measurements need to provide the centroid and width of the GMR to better than 150 keV, after taking into account possible uncertainties in the continuum. Significant systematic errors between differing measurements must be removed. The strength distribution in light nuclei must be mapped over a wide energy range. It will be worthwhile to carry out RPA calculations of the GMR with effective interactions that reproduce the ground-state properties of nuclei and the strength distribution of the GMR for light as well as heavy nuclei.

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