RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 31, NUMBER 6 JUNE 1985

Probing heavy ion collisions with bremsstrahlung

Che Ming Ko,* G. Bertsch, and J. Aichelin
Joint Institute for Heavy Ion Research, Oak Ridge National Laboratory,
Oak Ridge, Tennessee 37831

(Received 27 March 1985)

We calculate bremsstrahlung spectra for intermediate energy heavy ion collisions to see the sensitivity of photon cross sections to reaction dynamics. Modeling the collisions with the intranuclear cascade, we find a clear collective quadrupole bremsstrahlung signal only for heavier nuclei, $A > 40$. Maximum sensitivity to dynamics is likely to be for photons in the range of 10–30 MeV. For higher energy photons or lighter nuclei, there is a large background coming from the incoherent dipole component of the bremsstrahlung.

Most observables in heavy ion collisions involve the strongly interacting particles and are therefore more sensitive to the final stages of the collision than to the initial stages. To study questions such as the equation of state of nuclear matter which affects only the initial development of the collision, it is desirable to use weakly interacting probes such as photons. Kapusta[1] suggested probing the dynamics of high-energy heavy ion collisions by the bremsstrahlung photons which are created during the initial slowing down of the colliding nuclei. For the ultrarelativistic domain, a theoretical study was made recently by Bjorken and McLerran.[2] Experiments carried out so far on relativistic collisions have been inconclusive;[3] one problem is that photons from π^0 decay make a severe background in the higher-energy photon spectrum. In the intermediate energy domain, preliminary data have been reported by Grosse *et al.*[4] for the reaction $^{12}C+^{12}C$ and $^{12}C+^{238}U$ at an incident energy of 84 MeV/nucleon. For the theory of bremsstrahlung at intermediate energy, Vasak *et al.*[5] have constructed a phenomenological model based on classical trajectories. In this Rapid Communication, we shall apply the intranuclear cascade model to this energy domain. Our object is to determine the sensitivity of the photon spectra to the collision dynamics and suggest the most favorable experiments for such bremsstrahlung studies.

The formula for bremsstrahlung may be derived from classical radiation theory. The differential probability to radiate a photon from a source of charge density $\mathbf{j}(\mathbf{r},t)$ is given by

$$\frac{d^2P}{d\omega\, d\Omega}=\frac{\omega}{4\pi^2}\sum_{\lambda}^{2}\left|\int_{-\infty}^{+\infty} dt e^{-i\omega t}\int d^3r e^{i\mathbf{k}\cdot\mathbf{r}}\mathbf{j}(\mathbf{r},t)\cdot\boldsymbol{\epsilon}_\lambda\right|^2 . \quad (1)$$

In the above equation ω, $\mathbf{k}$, and $\boldsymbol{\epsilon}_\lambda$ denote the frequency, the wave vector, and the polarization vectors of the photon, respectively. We have used natural units $\hbar=c=1$. Before discussing the numerical evaluation of Eq. (1) it is useful to understand the structure of the result in limiting cases.

Consider first a collision between individual nucleons of the projectile and the target. Initially they move along the z axis towards each other, and after a collision (assumed to be instantaneous in the intranuclear cascade), they move apart at constant velocity. The integral over current separates into two parts, before and after the collision. We add the amplitudes of all the protons and assume that the final velocities are isotropically distributed to throw away the post-collision integrals. In the long wavelength limit, this gives the following expression for the bremsstahlung probability

$$\frac{d^2P}{d\omega\, d\Omega}=\frac{\alpha Z_{\text{eff}}^2 v^4\sin^2(2\theta)}{4\pi^2\omega(1-v^2\cos^2\theta)^2} . \quad (2)$$

Here v is the velocity of the proton in the c.m. frame, α is the fine structure constant, and θ is the angle between $\mathbf{k}$ and $\mathbf{v}$. The quantity Z_{eff} denotes the effective number of protons in each nucleus that interact. Averaging the collision over the impact parameter, we obtain $Z_{\text{eff}}\sim Z/5$ for the collision between symmetric systems with a charge Z. We see from Eq. (2) that for nonrelativistic velocities the angular distribution has the characteristics of quadrupole radiation. All previous studies on bremsstrahlung are based on this coherent radiation field.

Although collisions between protons always lead to quadrupole radiation, the proton-neutron collisions can produce incoherent dipole radiation as well. There will be a net current in the z direction if unequal numbers of protons from the projectile and target are involved in the collision. Also, the scattered protons will have a net current in the final state, of fluctuating magnitude and direction. Assuming the scattered protons velocities to be uncorrelated, we obtain in the long wavelength limit an additional contribution to the bremsstrahlung given by

$$\frac{d^2P}{d\omega\, d\Omega}\cong\frac{\alpha Z_{\text{eff}} v^2(c\sin^2\theta+d)}{4\pi^2\omega} , \quad (3)$$

which obviously has a dipolar shape. The coefficients c and d may be estimated assuming all of the protons interact independently. For $^{40}Ca+^{40}Ca$ at impact parameter $b=R$, this gives $c=0.72$ and $d=\frac{4}{3}$, in good agreement with the computed angular distribution from the intranuclear cascade model. If there are correlations between the protons in the initial state, as, for example, Pauli correlations,[6] the coefficient c may be reduced. Comparing Eqs. (2) and (3), we see that if the collision is relativistic ($v\approx 1$), the coherent contribution dominates over the incoherent by the factor Z_{eff}. For intermediate energy collisions, however, with $v<1$, the quadrupole component is reduced by a factor v^2, so the incoherent dipole radiation will be important for collisions between ions with low Z.

Equations (2) and (3) are valid only in the long wavelength, low-frequency limit. When the photon frequency becomes of the order of the inverse collision time, the $1/\omega$ spectrum should change to a more rapidly decreasing function. Counterbalancing this is the fact that the fluctuations in the currents during the collision time can contribute to high-frequency radiation, but interfere destructively at low frequency. Thus the interesting information about the heavy ion collision dynamics comes from deviations from the simple formulas above.

In order to study the bremsstrahlung process in detail we have carried out a calculation using the intranuclear cascade model of Bertsch and Cugnon.[7] In this model, the entire dynamics comes from nucleon-nucleon collisions, which are assumed to behave as in the free scattering cross section. The model neglects the Pauli momentum of the nucleus, Pauli blocking of the collisions, and the mean field acceleration of the nucleons. We feel that these effects will be small enough at incident energies of 100 MeV/nucleon so that a calculation omitting them will still provide a useful orientation.

From the intranuclear cascade (INC) code, we obtain the positions and velocities of the nucleons as a function of time. The charge current density is given by the sum

$$\mathbf{j}(\mathbf{r},t)=e\sum_{n}^{Z_p,Z_T}\delta(\mathbf{r}-\mathbf{r}_n(t))\mathbf{v}_n(t)\ . \qquad (4)$$

The amplitude for emitting a photon is then determined by the integral

$$A_\lambda=e\sum_{n}^{Z_p,Z_T}\int_{-\infty}^{+\infty}dt e^{-i\omega t+i\mathbf{k}\cdot\mathbf{r}_n(t)}\mathbf{v}_n(t)\cdot\boldsymbol{\epsilon}_\lambda\ . \qquad (5)$$

Since the velocity of a nucleon changes only at the instants when collisions occur Eq. (5) can be integrated as

$$A_\lambda=ie\sum_{n}^{Z_P,Z_T}\sum_{t_\alpha}e^{-i\omega t_\alpha+i\mathbf{k}\cdot\mathbf{r}_n(t_\alpha)}\left[\frac{v_n^{(+)}(t_\alpha)\cdot\epsilon_\lambda}{-\omega+k\cdot v_n^{(+)}(t_\alpha)}-\frac{v_n^{(-)}(t_\alpha)\cdot\epsilon_\lambda}{-\omega+k\cdot v_n^{(-)}(t_\alpha)}\right]\ , \qquad (6)$$

where $v^{(-)}$ and $v^{(+)}$ are velocities of the particle before and after each collision. A similar expression to Eq. (6) can be derived for the quantum mechanical bremsstrahlung amplitude in the soft-photon approximation. The only differences in the amplitude expression within the brackets are that $v_n^{(+)}$ is replaced by the velocity after photon is emitted, and the denominators contain the small terms $k^2/2m$. The cross section in the soft-photon approximation will differ also because the density of final nucleon states is evaluated at the final nucleon energy after the photon is emitted. We note that the soft-photon approximation is reasonably accurate, having an error of only 25% for 30 MeV photons emitted from 100 MeV NN collisions.[8]

We have performed numerical calculations for various symmetric systems at incident energies of 100 MeV/nucleon, using Eq. (6). The angular distribution is shown for photon energies of 10, 20, and 40 MeV in Fig. 1 by the solid, long-dashed, and dash-dotted curves, respectively. The angular distribution varies with both systems studied and photon energy. The coherent component of the bremsstrahlung can be isolated by treating all INC particles as protons, in which case the net current in the final state is zero. The cross sections for a 10 MeV photon obtained under this assumption are shown as dashed curves in the same figure. We see that the angular distribution becomes purely quadrupolar. The quadrupole component of the total photon radiation is only about 10% in the case of ^{16}O, and rises to about 70% in the case of ^{208}Pb. Thus the coherent quadrupole is only prominent for the heavier systems, in agreement with expectation from Eqs. (2) and (3) when $v=\frac{1}{4}$. We also find that the quadrupole component is enhanced if contributions from only low impact parameters are included. This might be achieved by suitable triggers on the experimental measurement. Since the coherent quadrupole radiation is more sensitive to the global dynamics of the collision, a central trigger would be very desirable to apply to the bremsstrahlung probe. A central trigger will also reduce the dipole radiation from target and projectile excitation.

The angle integrated energy spectra of the bremsstrahlung are shown in Fig. 2. All spectra show the general $1/\omega$ behavior for photon energies below 20 MeV. At high energies, the spectrum does not show the faster drop we expected, but instead flattens out. This is due to an additional contribution to the incoherent dipole bremsstrahlung, arising from the fluctuations in the current during the collision process. However, the calculation is not reliable much above 20 MeV photon energy. The initial nucleon-nucleon

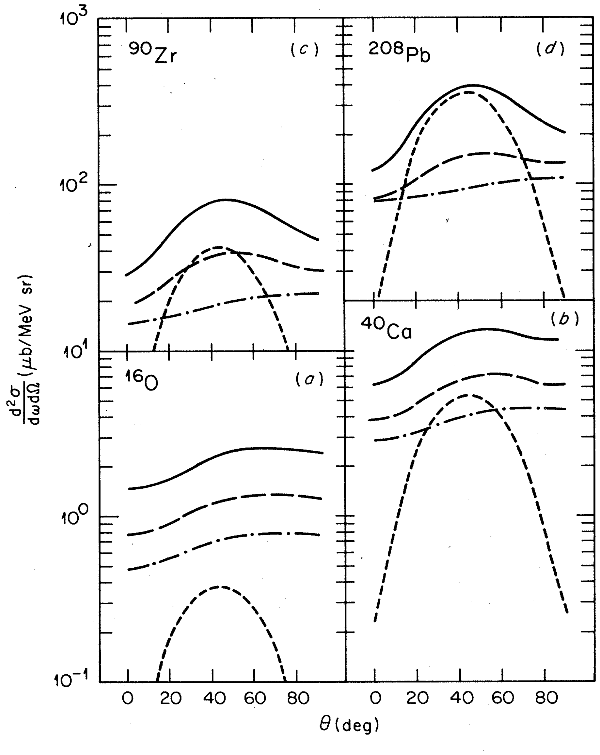

FIG. 1. Inclusive photon angular distributions for reactions (a) $^{16}\mathrm{O}+{}^{16}\mathrm{O}\rightarrow\gamma+X$, (b) $^{40}\mathrm{Ca}+{}^{40}\mathrm{Ca}\rightarrow\gamma+X$, (c) $^{90}\mathrm{Zr}+{}^{90}\mathrm{Zr}\rightarrow\gamma+X$, and (d) $^{208}\mathrm{Pb}+{}^{208}\mathrm{Pb}\rightarrow\gamma+X$, all at incident energies 100 MeV/nucleon. Solid, long-dashed, and dash-dotted curves are from full INC calculations for photon energies 10, 20, and 40 MeV, respectively. The dashed curves are from proton INC calculations for photon energy 10 MeV, which artificially suppresses the dipole component. The statistical errors associated with the INC calculations are less than 10%.

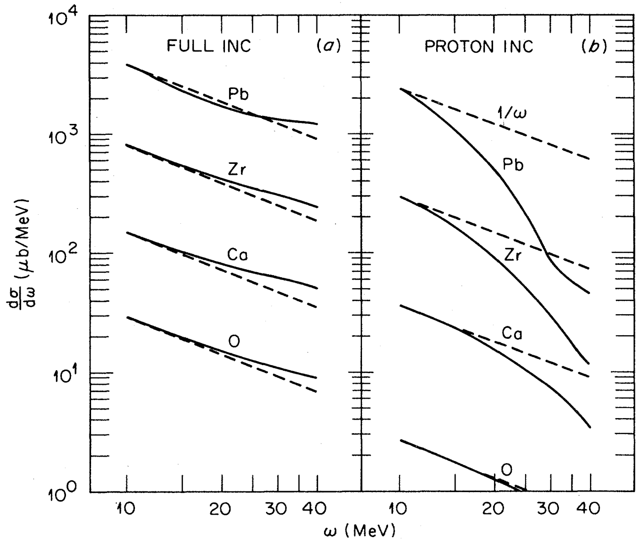

FIG. 2. Inclusive photon energy spectra for reactions in Fig. 1. Solid curves are from INC calculations, which may be compared with a $1/\omega$ behavior shown as the dashed lines.

pair energies are 50 MeV in the INC model (which has no Fermi momentum), which establishes a maximum photon energy, to be contrasted with Eq. (6) which allows all photon energies. Weighting the collisions according to the final state phase space, we find a reduction of 20% for 20 MeV photons and a factor of 5 for 40 MeV photons. This reduction would not be so servere if Fermi momentum is included; however, the Pauli blocking of the final state would also act to reduce the high-energy photons. Finally, the mean field can act either to increase or decrease the photon radiation, depending on frequency. The quadrupole component does not exhibit this flattening, since it is dependent on the coherence over many particles. Indeed, for the heavier systems, the quadrupole component drops faster than $1/\omega$. This is due to the finite time and spatial extensions of the collision process, which are the properties of the collision we wish to learn about. Thus the limitations of the INC model are at a higher frequency than is required for our purposes.

In conclusion, our calculation of bremsstrahlung in heavy ion collisons at nonrelativistic energies shows the necessity of using high-Z targets and projectiles to make the coherent part of the cross section dominant. The coherent radiation in the energy range of 20 MeV is sensitive to the time scale of slowing down of the two nuclei, and so in principle carries information about the high-density dynamics. However, we have not yet calculated with a sophisticated enough model to demonstrate that such a sensitivity really exists. In that regard it will be of great interest to perform similar calculations for heavy ion collisions using the theory based on the Boltzmann-Uehling-Uhlenbeck equation, which includes mean field and Pauli blocking effects.[9] The use of the BUU equation would also automatically include dipole contributions due to the isovector part of the mean field. Most of the cross section for lighter nuclei arises from an incoherent dipole component. We find that the photon spectrum extends to very high frequencies, but most of this is an artifact of the INC model. In a more realistic treatment, the bremsstrahlung rate would be determined by the empirical NN scattering.

We acknowledge support by the University of Tennessee, the National Science Foundation under Grants No. PHY-84-13287 and No. PHY-84-06676, and the Department of Energy under Contract No. DE-AC05-840R21400.

---

[*] Permanent address: Department of Physics and Cyclotron Institute, Texas A&M University, College Station, TX 77843.

[1] J. I. Kapusta, Phys. Rev. C **15**, 1580 (1977).

[2] J. D. Bjorken and L. McLerran, Phys. Rev. D **31**, 63 (1985).

[3] M. P. Budiansky, S. P. Ahlen, G. Tarle, and P. B. Price, Phys. Rev. Lett. **49**, 361 (1982).

[4] E. Grosse, in The Fifth Adriatic International Conference on Nuclear Physics, Hvar, 1984 (to be published).

[5] D. Vasak, W. Greiner, B. Müllen, T. Stahl, and M. Uhlig, Nucl. Phys. **A428**, 291C (1984).

[6] G. F. Bertsch, Phys. Rev. Lett. **46** 472 (1981).

[7] G. F. Bertsch and J. Cugnon, Phys. Rev. C **24**, 2514 (1981).

[8] P. Signell, in *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Table IX, p. 282.

[9] G. F. Bertsch, H. Kruse, and S. Das Gupta, Phys. Rev. C **29**, 673 (1984); J. Aichelin and G. F. Bertsch, Phys. Rev. C (to be published).