(\(^3\)He, d) reaction to bound and quasibound levels in \(^{59}\)Cu\(^\dagger\)

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The \(^{58}\)Ni(\(^3\)He, d)\(^{59}\)Cu reaction has been studied at bombarding energies of 35 and 39 MeV. Excitation energies and angular distributions were measured for levels up to 9.2 MeV in excitation. Distorted-wave-Born-approximation calculations were performed using resonance form factors for the proton-unstable levels \((E_x \geq \sim 3.4 \text{ MeV})\) in order to make \(l\) assignments and to extract the spectroscopic factors. Twelve levels in \(^{59}\)Cu are identified as possible analogs of states in \(^{58}\)Ni. A particle-core-coupling model is used to predict the properties of \(^{59}\)Cu and fair agreement with the experiment is obtained.

\[
\begin{bmatrix}
\text{NUCLEAR REACTIONS, NUCLEAR STRUCTURE} \\
\text{\(^{58}\)Ni(\(^3\)He, d) \(E = 35.0, 39.1 \text{ MeV}\) measured \(\sigma(\theta)\), \(^{59}\)Cu levels, deduced \(l, S\); identified analog states, calculated \(l, \tau, S\), particle-core-coupling model.}
\end{bmatrix}
\]

I. INTRODUCTION

The structure of \(^{58}\)Cu has been studied by several groups both experimentally and theoretically. The \((\(^3\)He, d), (\(\alpha, l\)),\) and \((d, n)\) reactions have been used extensively\(^{1-6}\) to obtain spectroscopic information on \(^{58}\)Cu. Analyses of these results were generally limited to levels of excitation \(\leq 4 \text{ MeV}\), because the states above 3.4 MeV are proton unstable, and therefore distorted-wave-Born-approximation (DWBA) calculations could not be performed in the usual manner. However, these studies yielded important spectroscopic information on the low-lying states and the excitation energies of many higher-lying states. In addition, Nelson et al.,\(^7\) have used the \(^{58}\)Ni(\(^3\)He, \(d\gamma\))\(^{59}\)Cu reaction to determine \(J^\pi\) and obtain mean lifetimes of levels in \(^{58}\)Cu below 3.6 MeV. Considerable information on unbound levels in \(^{58}\)Cu has been obtained\(^{8-10}\) by proton elastic scattering on \(^{58}\)Ni.

Several levels have been identified as possible analogs of levels in \(^{58}\)Ni and their properties measured. Investigations\(^{11-13}\) of the \(\gamma\) decay of the levels at 6.842, 6.889, and 6.903 MeV in \(^{58}\)Cu populated with the \(^{58}\)Ni(\(p, \gamma\))\(^{59}\)Cu reaction have led to the determination of spin assignments for a few levels above 3.4 MeV in excitation.

There has been considerable speculation\(^1-2\) about the isospin character of many of the unbound levels observed in proton stripping reactions. The positions and relative strengths were used as criteria for identifying \(^{58}\)Cu states as analogs of states in \(^{58}\)Ni; however, spectroscopic factors and \(l\) assignments were not available. The most reliable identification of isobaric analog states (IAS) has been done utilizing the elastic scattering\(^6,10\) and radiative capture data\(^{11-13}\) where four states have been identified as analogs on the basis of energy, \(J^\pi\), and spectroscopic factors.

Unfortunately the study of levels just above threshold is not practical with elastic proton scattering due to the large Coulomb barrier, while even at higher energies states with high \(l\) values will be suppressed. The existing elastic scattering data provide no information on unbound states below \(E_x = 5.2 \text{ MeV}\) and could easily miss states having higher penetrability above this energy. Stripping reactions however are not very sensitive to the penetrability, as the yield (for a given \(l\)) is essentially proportional to the single-particle strength of the state with only a slight excitation energy dependence. Techniques have now been developed\(^14,15\) to analyze stripping reactions populating unbound levels; however most of the published data were taken at relatively low energies where DWBA analysis, particularly of higher excited states, is at best suspect due to the low energy of the outgoing particle. Thus the \(^{58}\)Ni(\(^3\)He, d)\(^{59}\)Cu reaction was studied at 35 and 39 MeV incident energies to obtain additional nuclear structure information about the higher excited states in \(^{59}\)Cu. Additionally, utilizing the above mentioned DWBA techniques, the existing data for population of unbound levels in \(^{59}\)Cu have been analyzed where practical.

II. EXPERIMENTAL PROCEDURE

The experimental details are similar to those described previously.\(^16\) A 305 \(\mu\)g/cm\(^2\) self-supporting Ni foil, enriched to \(\sim 99\%\) in \(^{58}\)Ni, was bombarded with 35.0 and 39.1 MeV \(^3\)He ions accelerated by the Texas A\&M University cyclotron. Two silicon detector telescopes spaced 5° apart were used to reduce data acquisition time and
selected data points were checked by measurements with both systems. The detector telescopes consisted of 2 mm (ΔE) and 3 mm (E) detectors for both stacks. A typical 3He spectrum is shown in Fig. 1. The overall resolution was about 50 keV full width at half maximum. Due to the high density of levels in the high excitation region, a multipeak fitting program, described previously, was used in analyzing the data.

III. DISTORTED-WAVE-BORN-APPROXIMATION CALCULATIONS

A. Bound states

DWBA calculations were performed using the computer code DWUCK. Single-particle wave functions, used as form factors for stripping to bound states in 59Cu, were generated in the usual manner by varying the depth of the Woods-Saxon well. The calculations were done both using the zero-range approximation (ZR) and utilizing finite-range and nonlocal (FRNL) corrections. The optical model, form factor, and FRNL parameters are listed in Table I. The calculated and experimental cross sections are related by

\[ \left( \frac{d\sigma}{d\Omega} \right)_{DWBA} = C^{2S+1/2} \left( \frac{d\sigma}{d\Omega} \right)_{DF}, \]

where J is the transferred angular momentum, N is the normalization constant determined from the internal structure of the projectile, and C^2S is the spectroscopic factor. We have used values of N = 4.42 and 1.65 for the (3He, d) and (d, n) reactions, respectively.

B. Quasibound states

Levels above 3.4 MeV excitation energy are proton unstable and therefore the usual DWBA computer codes cannot be used to analyze the data. The DWBA form factors needed for these levels were generated by varying the depth of the Woods-Saxon well to produce a resonance at the proper energy having the correct (l, J). Zero-range DWBA calculations were performed in the same manner as for the bound states; however, a large cutoff radius (200 fm) is required for convergence of the radial integrals if the exterior amplitude of the form factor is large (>0.01 times the interior amplitude). This method is essentially equivalent to that of Vincent and Fortune and has been

| Table I. Optical model and finite-range nonlocal (FRNL) parameters used in DWBA calculations (in MeV fm). |

<table>
<thead>
<tr>
<th>Particle</th>
<th>(E_{3\text{He}})</th>
<th>(V)</th>
<th>(r_0)</th>
<th>(a)</th>
<th>(W)</th>
<th>(4W_0)</th>
<th>(r_1)</th>
<th>(a_1)</th>
<th>(r_c)</th>
<th>(\beta)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^3\text{He}) &amp; 39 &amp; 169.5 &amp; 1.15 &amp; 0.752 &amp; 22.09 &amp; &amp; 1.57 &amp; 0.787 &amp; 1.40 &amp; 0.20</td>
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<tr>
<td>(d) &amp; 39 &amp; 103.8 &amp; 1.048 &amp; 0.825 &amp; &amp; &amp; 47.52 &amp; 1.252 &amp; 0.888 &amp; 1.30 &amp; 0.54</td>
<td></td>
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</tr>
<tr>
<td>(^3\text{He}) &amp; 35 &amp; 162.0 &amp; 1.193 &amp; 0.696 &amp; 20.3 &amp; &amp; 1.336 &amp; 0.901 &amp; 1.40 &amp; 0.50</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>(d) &amp; 35 &amp; 96.83 &amp; 1.081 &amp; 0.814 &amp; &amp; &amp; 51.6 &amp; 1.295 &amp; 0.784 &amp; 1.30 &amp; 0.54</td>
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</tr>
<tr>
<td>(p) &amp; 1.20 &amp; 0.65 &amp; &amp; &amp; &amp; &amp; &amp; &amp; &amp; 0.85</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

\(a\) Nonlocal parameters used in DWUCK.

\(b\) Finite-range parameters.

\(c\) Reference 19.

\(d\) Reference 20.

\(e\) Reference 21.

\(f\) Reference 22.
TABLE II. Summary of spectroscopic factors for bound states of $^{59}$Cu.

<table>
<thead>
<tr>
<th>$^{59}$Cu+</th>
<th>$E$ (MeV ± keV)</th>
<th>$l_p$</th>
<th>$J^\pi$</th>
<th>$\langle \text{He}, d \rangle_{59}$</th>
<th>$\langle 2J+1 \rangle C^2S_p$</th>
<th>Avg. (FRNL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{58}$Ni</td>
<td>0.0</td>
<td>1</td>
<td>$^3_2$</td>
<td>1.89</td>
<td>2.1</td>
<td>1.28</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>0.491±5</td>
<td>1</td>
<td>$^3_2$</td>
<td>0.59</td>
<td>1.0</td>
<td>0.72</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>0.913±7</td>
<td>3</td>
<td>$^3_2$</td>
<td>2.85</td>
<td>3.33</td>
<td>2.62</td>
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<tr>
<td>$^{59}$Cu</td>
<td>1.399±7</td>
<td>3</td>
<td>$^3_2$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.24</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>2.323±7</td>
<td>1</td>
<td>$^3_2$</td>
<td>0.29</td>
<td>0.25</td>
<td>0.17</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>2.710±7</td>
<td>3</td>
<td>$^3_2$</td>
<td>0.15</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>3.047±7</td>
<td>4</td>
<td>$^3_2$</td>
<td>3.96</td>
<td>2.94</td>
<td>2.27</td>
</tr>
<tr>
<td>$^{59}$Cu</td>
<td>3.137±7</td>
<td>1</td>
<td>$^3_2$</td>
<td>0.32</td>
<td>0.22</td>
<td>0.16</td>
</tr>
</tbody>
</table>

* Present work.
* $J^\pi$ assignments taken from Ref. 7.
* DWBA analysis using the zero-range approximation.
* Reference 3.
* Reference 5.
* Average of the $\langle 2J+1 \rangle C^2S_p$ obtained from the 35 and 39 MeV data using FRNL.

FIG. 2. Angular distributions for the $^{58}$Ni ($^3\text{He}, d$) $^{59}$Cu reaction leading to bound states. The excitation energies are indicated in MeV. The errors shown are statistical only. The curves are FRNL DWBA predictions for the $l$ transfer indicated.

FIG. 3. Angular distributions for the $^{58}$Ni ($^3\text{He}, d$) $^{59}$Cu reaction leading to bound states. See Fig. 2 caption.
TABLE III. Summary of experimental results for states of $^{59}$Cu with $E_x \approx$ 3.4 MeV.

<table>
<thead>
<tr>
<th>$^{59}$Cu * (MeV + keV)</th>
<th>$\ell_\nu$</th>
<th>$J^\pi$</th>
<th>$\langle \sum_{3}He,d \rangle$</th>
<th>$\langle \ell',\eta \rangle$</th>
<th>$E_x$ (MeV)</th>
<th>$I_p$</th>
<th>$G_{1\nu}^\pi$</th>
<th>Nuclear Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4.44)</td>
<td>3.582 ± 8</td>
<td>2</td>
<td>$\frac{3}{2}^+$</td>
<td>1.05</td>
<td>1.22</td>
<td>1.34</td>
<td>3.435 ± 12</td>
<td></td>
</tr>
<tr>
<td>(5.62)</td>
<td>3.707 ± 12</td>
<td>2</td>
<td>$\frac{3}{2}^+$</td>
<td>(0.12)</td>
<td>0.17</td>
<td></td>
<td>3.749 ± 8</td>
<td>2</td>
</tr>
<tr>
<td>(6.62)</td>
<td>3.737 ± 8</td>
<td>1</td>
<td>$\frac{3}{2}^-$</td>
<td>(0.12)</td>
<td>3.663</td>
<td>$\frac{1}{2}^+$</td>
<td>3.671 ± 25</td>
<td></td>
</tr>
<tr>
<td>(4.06)</td>
<td>3.897 ± 7</td>
<td>1</td>
<td>$\frac{3}{2}^-$</td>
<td>0.45 ^{2}</td>
<td>3.905</td>
<td>$\frac{3}{2}^+$</td>
<td>3.916 ± 8</td>
<td>1</td>
</tr>
<tr>
<td>(4.06)</td>
<td>4.002 ± 7</td>
<td>1</td>
<td>$\frac{3}{2}^-$</td>
<td>0.25</td>
<td>0.27</td>
<td>0.31</td>
<td>4.007 ± 6</td>
<td>1</td>
</tr>
<tr>
<td>(5.61)</td>
<td>4.116 ± 7</td>
<td>3</td>
<td>$\frac{3}{2}^-$</td>
<td>0.14</td>
<td>4.131 ^{h}</td>
<td></td>
<td>4.116 ± 6</td>
<td>(2)</td>
</tr>
<tr>
<td>(5.61)</td>
<td>4.261 ^{d}</td>
<td>3</td>
<td>$\frac{3}{2}^-$</td>
<td>0.05</td>
<td></td>
<td></td>
<td>4.267 ± 8</td>
<td></td>
</tr>
<tr>
<td>(5.61)</td>
<td>4.307 ± 7</td>
<td>3</td>
<td>$\frac{3}{2}^-$</td>
<td>1.05 ^{2}</td>
<td>1.36</td>
<td></td>
<td>4.312 ± 6</td>
<td>3</td>
</tr>
<tr>
<td>(4.70)</td>
<td>4.358 ± 10</td>
<td>1</td>
<td>$\frac{3}{2}^+$</td>
<td>0.30 ^{2}</td>
<td>0.29</td>
<td></td>
<td>4.362 ± 6</td>
<td>1</td>
</tr>
<tr>
<td>(4.77)</td>
<td>4.538 ± 10</td>
<td>3</td>
<td>$\frac{3}{2}^-$</td>
<td>0.09</td>
<td>0.14</td>
<td>(0.20)</td>
<td>4.539 ^{h}</td>
<td>4.550 ± 15</td>
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<tr>
<td>(4.77)</td>
<td>4.828 ± 10</td>
<td>3</td>
<td>$\frac{3}{2}^-$</td>
<td>0.15</td>
<td>(4.689 ^{h})</td>
<td></td>
<td>4.720 ± 15</td>
<td>4.777 ± 15</td>
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<tr>
<td>(6.30)</td>
<td>5.058 ± 10</td>
<td>1</td>
<td>$\frac{3}{2}^+$</td>
<td>0.08</td>
<td>0.08</td>
<td></td>
<td>5.063 ± 15</td>
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</tr>
<tr>
<td>(6.30)</td>
<td>5.234 ± 10</td>
<td>1</td>
<td>$\frac{3}{2}^+$</td>
<td>0.18</td>
<td>0.20</td>
<td>0.18</td>
<td>5.245 ± 15</td>
<td>5.321 ± 15</td>
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<tr>
<td>(6.35)</td>
<td>5.681 ± 10</td>
<td>3</td>
<td>$\frac{1}{2}^-$</td>
<td>0.14</td>
<td>0.20</td>
<td></td>
<td>5.659 ± 15</td>
<td></td>
</tr>
<tr>
<td>(6.35)</td>
<td>5.856 ± 10</td>
<td>3</td>
<td>$\frac{1}{2}^-$</td>
<td>0.15</td>
<td>0.84</td>
<td></td>
<td>5.858 ± 15</td>
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<tr>
<td>(6.35)</td>
<td>5.935 ± 10</td>
<td>1</td>
<td>$\frac{1}{2}^-$</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.11)</td>
<td>(4)</td>
<td>$\frac{3}{2}^+$</td>
<td>(0.50)</td>
<td></td>
<td></td>
<td></td>
<td>6.124 ± 20</td>
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<tr>
<td>(6.11)</td>
<td>6.202 ± 10</td>
<td>(3)</td>
<td>$\frac{3}{2}^+$</td>
<td>0.41</td>
<td>0.57</td>
<td></td>
<td>6.204 ± 20</td>
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<tr>
<td>(6.11)</td>
<td>(4)</td>
<td>$\frac{3}{2}^+$</td>
<td>0.48</td>
<td>1.05</td>
<td></td>
<td></td>
<td>6.314 ± 20</td>
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<tr>
<td>(6.35)</td>
<td>(4)</td>
<td>$\frac{3}{2}^+$</td>
<td>(0.07)</td>
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<tr>
<td>(6.35)</td>
<td>(1)</td>
<td>$\frac{3}{2}^+$</td>
<td>(0.13)</td>
<td></td>
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</tr>
<tr>
<td>(6.11)</td>
<td>6.513 ± 12</td>
<td>3</td>
<td>$\frac{1}{2}^-$</td>
<td>(0.21)</td>
<td>0.28</td>
<td></td>
<td>6.525 ± 20</td>
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</tr>
<tr>
<td>(6.73)</td>
<td>6.840 ± 12</td>
<td>4</td>
<td>$\frac{3}{2}^+$</td>
<td>0.39</td>
<td></td>
<td></td>
<td>6.740 ± 20</td>
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<tr>
<td>(6.81)</td>
<td>6.917 ± 12</td>
<td>4</td>
<td>$\frac{5}{2}^+$</td>
<td>0.70</td>
<td>0.95 ^{2}</td>
<td></td>
<td>6.902 $^{5}f, g, i$</td>
<td>6.910 ± 20</td>
</tr>
<tr>
<td>(7.20)</td>
<td>7.116 ± 12</td>
<td>2</td>
<td>$\frac{5}{2}^+$</td>
<td>0.13</td>
<td>0.16</td>
<td></td>
<td>7.128 ± 20</td>
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<tr>
<td>(7.26)</td>
<td>7.155 ± 15</td>
<td>(2)</td>
<td>$\frac{5}{2}^+$</td>
<td>(0.12)</td>
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<td>(7.26)</td>
<td>7.397 ± 15</td>
<td>(2)</td>
<td>$\frac{5}{2}^+$</td>
<td>(0.13)</td>
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<td>(7.26)</td>
<td>(4)</td>
<td>$\frac{5}{2}^+$</td>
<td>(0.14)</td>
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<tr>
<td>(7.68)</td>
<td>8.103 ± 20</td>
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<td>$\frac{5}{2}^+$</td>
<td>0.09</td>
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</table>
TABLE III (Continued)

<table>
<thead>
<tr>
<th>$^{58}$Cu* (MeV + keV)</th>
<th>$l_p$</th>
<th>$J^e$</th>
<th>$^{(3}\text{He},d)^{56}$Cu</th>
<th>$G_{ij}$</th>
<th>$(d',n)$</th>
<th>$(p',\gamma)$</th>
<th>$E_x$ (MeV)</th>
<th>$J^i$</th>
<th>$E_x$ (MeV + keV)</th>
<th>$l_p$</th>
<th>$G_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.169 ± 20</td>
<td>2</td>
<td>$\frac{1}{2}^+$</td>
<td>39 MeV</td>
<td>0.12</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.223 ± 20</td>
<td>2</td>
<td>$\frac{3}{2}^+$</td>
<td>39 MeV</td>
<td>0.11</td>
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<td></td>
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</tr>
<tr>
<td>8.532 ± 20</td>
<td>4</td>
<td>$\frac{5}{2}^+$</td>
<td>10 MeV</td>
<td>0.13</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.252 ± 20</td>
<td>4</td>
<td>$\frac{5}{2}^+$</td>
<td>10 MeV</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

$^a$ $G_{ij} = (2J + 1)C^2_{ij}$.  
$^b$ Reference 23.  
$^c$ Present work.  
$^d$ Present analysis of the data in Ref. 4.  
$^e$ $J^e$ values are those which seem most plausible on a shell-model basis; no assignments have been made.  
$^f$ Reference 13.  
$^g$ Analysis of the experimental data of Blair and Armstrong (Ref. 1) gave $(2J + 1)C_{ij} = 0.62$, 1.4, 0.31, and 0.85 for the 8.88, 4.30, 4.36, and 6.86 MeV states, respectively. They did not resolve the two levels at ~6.9 MeV.  
$^h$ Reference 11.  
$^i$ Reference 12.

FIG. 4. Angular distributions for the $^{58}$Ni($^{3}\text{He},d)^{56}$Cu reaction leading to quasibound states. The excitation energies are indicated in MeV. The curves are DWBA predictions for the $l$ transfer indicated obtained as described in the text.

FIG. 5. Angular distributions for the $^{58}$Ni($^{3}\text{He},d)^{56}$Cu reaction leading to quasibound states. See Fig. 4 caption.
are taken from the $^3\text{He},d\,^5\text{Cu}$ work of Nelson et al.\textsuperscript{7} Our energy locations, $l$ values, and spectroscopic factors obtained using ZR DWBA agree reasonably well with the previous work (which also used ZR DWBA) for all the states. The fits obtained with FRNL calculations were slightly better and the spectroscopic factors decreased somewhat.

B. Quasibound states

A summary of the results obtained in the present study for levels above 3.4 MeV in $^{56}\text{Ni}$ is given in Table III and compared with existing information from the literature. Weak levels observed only at a few angles are shown in parentheses. For $E_x \geq 7.12$ MeV only the stronger states were analyzed. Many weaker states obviously present could not be resolved and are not included in the table. Angular distributions and DWBA predictions for the 35 MeV data are shown in Fig. 4. The DWBA fits to the 39 MeV data are shown in Figs. 5 and 6. In addition to analyzing the present $^3\text{He},d$ data, we also analyzed states above 3.4 MeV in excitation populated in the $(d,n)$ reaction of Marusak\textsuperscript{4} with $E_d=10$ MeV and the $^3\text{He},d$ reaction of Blair and Armstrong\textsuperscript{4} with $E_{^3\text{He}}=22$ MeV.

A fit to the 6.917 MeV state excited in the Blair and Armstrong work is also shown in Fig. 6. Representative fits to the Marusak data for the $(d,n)$ reaction for four different $l$ transfers are shown in Fig. 7.

In previous work, spectroscopic factors and $l$-value assignments for some of the levels between 3.4 and 4.4 MeV excitation were obtained from proton transfer reaction studies by comparing with DWBA calculations performed assuming the states

![Figure 6](image-url)

**FIG. 6.** Angular distributions for the $^{58}\text{Ni}(^3\text{He},d)^{56}\text{Cu}$ reaction leading to the possible analogs of $^{58}\text{Ni}$. See Fig. 4 caption.

quite successful for the $^3\text{He},d$ reaction to unbound states in several nuclei.\textsuperscript{14,16} It has also been demonstrated\textsuperscript{16} that the same technique is adequate for analog states; that is, no special treatment of the form factor for the analog state is necessary to obtain reliable spectroscopic information.

IV. DISCUSSION OF RESULTS

A. Bound states

A summary of the experimental results for levels below 3.4 MeV in excitation is given in Table II. Angular distributions and DWBA predictions are shown in Figs. 2 and 3 for the 35 and 39 MeV data, respectively. The data are fitted reasonably well. The spectroscopic factors obtained in the present study are compared with previous work\textsuperscript{8-10} in Table II. The $J^+$ assignments

![Figure 7](image-url)

**FIG. 7.** Angular distributions for the $^{58}\text{Ni}(d,n)^{56}\text{Cu}$ reaction leading to quasibound states. The experimental data are from Ref. 4. See Fig. 4 caption.
were bound by a small amount. These results are generally in good agreement with the values we obtain as can be seen in Table III.

There is excellent agreement on \( l \) assignments for all the levels from the analysis of the four different sets of data except for the 4.116 and 6.202 MeV levels. An \( l = 3 \) assignment is favored for the 4.116 MeV level from the \((^3\text{He},d)\) reaction data whereas an \( l = 1 \) assignment is favored from the \((d,n)\) data. Some \( l = 1 \) contribution cannot be ruled out in our data. In the case of the 6.202 MeV level, the 35 MeV data are fitted equally well by \( l = 3 \) and \( l = 4 \) transfers, whereas the 39 MeV data are fitted better by an \( l = 3 \) transfer and the \((d,n)\) reaction data are fitted better by an \( l = 4 \) transfer. However, the analysis of the \((d,n)\) reaction leading to such highly excited levels is not very trustworthy as the outgoing neutron is left with so little energy that the usual DWBA is probably not applicable here. This may be the reason that the spectroscopic factors extracted from the \((d,n)\) reaction for levels above 5.3 MeV are very high as compared to those obtained from the analyses of the 35 and 39 MeV \((^3\text{He},d)\) reaction data. The present analysis shows that except for the two levels at 3.58 and 3.71 MeV, which are \( l = 2 \), all the levels up to 6 MeV in excitation correspond to \( l = 1 \) transfer (possible \( 2p_{3/2} \) and \( 2p_{1/2} \)) or \( l = 3 \) transfer (\( 1f_{5/2} \)). Most of the levels between 6.0 and 7.2 MeV are \( l = 4 \) (probably \( 1g_{9/2} \)), and at higher excitation both \( l = 2 \) and \( l = 4 \) states are excited.

Levels observed with the \((p, \gamma)\) reaction\(^1\) relevant to this work are also included in Table III. Unfortunately, many of the levels reported in these investigations are weakly excited in the present work and for these no meaningful comparison is possible. Arai, Ogawa, and Sato\(^1\) have reported many resonances in \(^{56}\text{Cu}\) using \(^{58}\text{Ni}(p,p)\) in the excitation region of 6.5 to 7.4 MeV.

We have calculated the single-particle laboratory widths for the strong resonances using an optical model\(^6\) with the same potentials as those used for the corresponding DWBA form factor. Using the experimental widths, we have extracted the values of \((2J + 1)C_S(T)\) for these resonances (Table IV). Our spectroscopic factors and \( l \) assignments for the \( ^\frac{3}{2}^+ \) states at 6.84 and 6.92 MeV (well resolved at 39 MeV, but not at 35 MeV) and the \( ^\frac{5}{2}^- \) state at 7.1 MeV are in agreement with \((p,p)\) results. The \( ^\frac{1}{2}^- \) states at 7.19 and 7.26 MeV are observed at a few angles but, as expected from their low \((p,p)\) spectroscopic factors, were too weak to obtain definitive angular distributions. The rest of the states would be too weak to observe in the present study.

### Table V. Spectroscopic factors for analog states.

<table>
<thead>
<tr>
<th>(^{58}\text{Ni}(p,p)) (MeV)</th>
<th>(^{58}\text{Ni}^* + 3.910) a (MeV)</th>
<th>(^{58}\text{Ni}^* ) (MeV)</th>
<th>(^{58}\text{Ni}^* ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.897</td>
<td>1 ( \frac{1}{2}^- )</td>
<td>0.52</td>
<td>3.910</td>
</tr>
<tr>
<td>4.307</td>
<td>3 ( \frac{1}{2}^- )</td>
<td>1.20</td>
<td>4.250</td>
</tr>
<tr>
<td>4.358</td>
<td>3 ( \frac{1}{2}^- )</td>
<td>0.30</td>
<td>4.375</td>
</tr>
<tr>
<td>4.828</td>
<td>1 ( \frac{1}{2}^- )</td>
<td>0.13</td>
<td>4.791</td>
</tr>
<tr>
<td>5.235</td>
<td>1 ( \frac{1}{2}^- )</td>
<td>0.19</td>
<td>5.308</td>
</tr>
<tr>
<td>5.681</td>
<td>3 ( \frac{1}{2}^- )</td>
<td>0.17</td>
<td>5.593</td>
</tr>
<tr>
<td>5.856</td>
<td>3 ( \frac{1}{2}^- )</td>
<td>0.15</td>
<td>5.861</td>
</tr>
<tr>
<td>6.513</td>
<td>3 ( \frac{1}{2}^- )</td>
<td>0.24</td>
<td>6.544</td>
</tr>
<tr>
<td>6.917</td>
<td>4 ( \frac{1}{2}^- )</td>
<td>0.70</td>
<td>6.970</td>
</tr>
<tr>
<td>8.22</td>
<td>2 ( \frac{1}{2}^- )</td>
<td>0.09</td>
<td>8.414</td>
</tr>
<tr>
<td>8.53</td>
<td>4 ( \frac{1}{2}^- )</td>
<td>0.13</td>
<td>8.605</td>
</tr>
<tr>
<td>9.25</td>
<td>4 ( \frac{1}{2}^- )</td>
<td>0.12</td>
<td>9.361</td>
</tr>
</tbody>
</table>

\(^a\) \( C_0 = (2J + 1)C_S \), averages of the spectroscopic factors given in Table III.

\(^b\) \( C_0 = (2J + 1)C_S \), averages of the spectroscopic factors obtained from Refs. 24–26.
4 MeV do not nearly exhaust all the strength of the 2p_{1/2}, 2p_{3/2}, 1f_{5/2}, and 1f_{7/2} orbitals however, so considerable T_{g} strength is expected to lie in the same excitation energy region as the T_{g} states. This makes it difficult to identify the T_{g} levels in ^{9}Cu. We have utilized the information on the energy locations, J^{*} assignments, and spectroscopic factors obtained in the present analysis and compared it to the information known in ^{59}Ni to identify some of the analog states in ^{59}Cu. This information is summarized in Table V and relevant angular distributions are shown in Figs. 4 and 6. The energy and J^{*} assignments for all the states listed here agree well with that expected from ^{59}Ni. These results generally agree with previous identification of the analogs, although several additional ones are located. The averages of the spectroscopic factors obtained for ^{59}Cu using different reactions (Table III) are compared with the averages of the spectroscopic factors for ^{59}Ni obtained from the studies of Fulmer et al., Cosman et al., and Chowdhury and Sen Gupta. For low-lying analog states, the spectroscopic factors agree well; however, for higher excited states, they are somewhat lower than those predicted using the ^{59}Ni data. The largest discrepancy exists for the 6.917 MeV state in ^{59}Cu which has been identified to be the analog of the $^3_2^+$, 3.06 MeV state in ^{59}Ni by (p,γ) studies. The observed spectroscopic factors for this state are only about 24% of the value expected from the ^{59}Ni data. In the recent study of ^{59}Ni(p,γ) by Arai et al., the g_{9/2} resonance at E_{γ} = 3.5446 MeV was identified as the analog of the 3.06 MeV ^{59}Ni state. Utilizing the technique of Thompson, Adams, and Robson, a spectroscopic factor [(2J+1)C^{3}S] of 1.5 is obtained compared to our value of 0.70. However, Arai et al. point out that they were unable to obtain reasonable criteria for determining the matching radius upon which the spectroscopic factor critically depends. As can be seen in Table IV, the spectroscopic factor extracted from the data of Arai et al. utilizing the simple optical model technique is in agreement with the (^3He,d) results.

V. STRUCTURE CALCULATIONS WITH THE PARTICLE-CORE-COUPLING MODEL

In a simple shell-model prescription, one would expect that for the Ni isotopes (Z = 28), the proton 1d_{5/2}, 2s_{1/2}, 1d_{5/2} and 1f_{7/2} subshells are completely filled and all other higher orbitals are empty. Therefore, the structure of low-lying levels in ^{59}Cu, where one proton has been added to the ^{59}Ni core should be simple and easy to describe. However, shell-model calculations of Singh et al. could not describe the structure of ^{59}Cu adequately. Using a unified model, Castel et al. have been successful in reproducing a few of the low-lying states. Larner has calculated the structure of N = 29 or Z = 29 nuclei using a simple core-coupling model with reasonable success but he did not do calculations for ^{59}Cu due to lack of enough experimental data at that time. We have used a similar model to predict the negative parity $^1_2^+$, $^3_2^+$, $^5_2^+$, $^7_2^+$, and $^5_2^+$ states of ^{59}Cu by coupling of the protons in 2p_{1/2}, 2p_{3/2}, and 1f_{7/2} orbitals with the 0^+ ground state and the first 2^+ state of ^{59}Ni. Details of this model are given elsewhere. The values of Hamiltonian parameters χ_1, χ_3, and η, defined in Ref. 17, were fixed to be 0.47 MeV fm^{-2}, 0.48 MeV fm^{-2}, and 0.28 MeV, respectively, in accordance with those used by Larner in his analysis of other Cu isotopes (A = 61, 63, and 65). The single-particle energies were taken as $\epsilon_1 = \epsilon(p_{1/2}) - \epsilon(p_{3/2}) = 0.85$ MeV and $\epsilon_2 = \epsilon(f_{5/2}) - \epsilon(p_{3/2}) = 1.15$ MeV.

\[ \text{FIG. 8. Comparison of the experimental levels with the theoretical predictions for} \ ^{59}\text{Cu. The} \ J^* \text{values are indicated on the right.} \]
Theoretical and experimental results for $^{50}$Cu up to 3.4 MeV in excitation are shown in Fig. 8 and the agreement is excellent for the first four levels with respect to both energy location and $J^*$ assignments. In this figure, we have also included as dashed lines the possible negative-parity levels that were not observed in the present study but were included in the decay scheme by Nelson et al.\(^7\) Although the comparison of these states with theory could be speculative, it appears that the observed $\frac{3}{2}^-$ state at 1.986 MeV corresponds to the $\frac{5}{2}^-$ state predicted at 1.43 MeV. The higher experimental states, namely the $\frac{3}{2}^-$, $\frac{7}{2}^-$, and $\frac{5}{2}^-$ states between 2.3 and 2.7 MeV in excitation agree in the order and the $J^*$ assignments with those predicted but are about 100 keV lower. The higher states are not shown as they are unbound to proton decay.

VI. CONCLUSIONS

The $^{\alpha}$(He, $d$) reaction at 35 and 39 MeV bombarding energies has been used to populate the proton states of $^{50}$Cu. The usual DWBA analysis was made for the states below 3.4 MeV to make $J^*$ assignments and to extract spectroscopic factors. The results obtained for these states in the present study agree very well with previous studies. At higher energy, we analyzed levels up to 9.25 MeV in excitation and many new levels have been identified. DWBA analysis of these proton-unstable states above 3.4 MeV was done for the present ($^{\alpha}$(He, $d$) reaction data, the $(d,n)$ reaction data of Marusak with $E_d=10$ MeV, and the ($^{\alpha}$(He, $d$) reaction work of Blair and Armstrong with $E_{\alpha}=22$ MeV. $J^*$ assignments and the spectroscopic factors were extracted and in general are in good agreement for the different reactions. The levels seen in the present study between $3.74 \leq E_x \leq 5.96$ MeV probably belong to $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbitals, whereas most of the levels at higher excitation energy belong to $1g_{9/2}$ and $2d_{5/2}$ orbitals as expected.

Twelve levels are identified as likely analogs of levels in $^{58}$Ni. The spectroscopic factors for most of the analogs are in agreement with parent state spectroscopic factors obtained from $^{58}$Ni($d$,$p$).\(^24\)\(^-\)\(^26\) However, the spectroscopic factor for the $g_{9/2}$ state at 6.917 MeV is about 0.70, much smaller than the expected value of 2.9. Our value is in agreement with the value of 0.57 we obtain using the $^{58}$Ni($p$,$p$) data of Arai et al.\(^10\)

A simple core-coupling model is used to predict the properties of low-lying levels in $^{50}$Cu. The calculated results are in good agreement with experimental data. Because the levels above 3.42 MeV are proton unbound, comparison is restricted to levels below this energy.

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