Cavity-QED-based quantum phase gate

M. Suhail Zubairy,[1,2] Moochan Kim,[1] and Marlan O. Scully[1,3]
[1]*Department of Physics and Institute for Quantum Studies, Texas A&M University, College Station, Texas 77843-4242, USA*
[2]*Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan*
[3]*Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*

(Received 25 February 2003; published 29 September 2003)

We describe a quantum phase gate in which the two qubits are represented by the photons in the two modes of the cavity field. The gate is implemented by passing a three-level atom in a cascade configuration through the cavity. The upper levels of the atom are resonant with one of the cavity modes whereas the lower levels are appropriately detuned from the other mode of the cavity. A π phase shift is introduced when there is one photon each in the two modes and the atom is initially in the ground state. We also discuss the one-bit unitary gate in such a system and discuss potential applications.

DOI: 10.1103/PhysRevA.68.033820 PACS number(s): 42.50.Gy, 03.67.−a, 42.50.Dv, 42.65.An

I. INTRODUCTION

Quantum computing [1] employs the principle of coherent superposition and quantum entanglement to solve certain problems much faster than on a conventional computer. The basic building blocks of a quantum computer are quantum logic gates. A universal quantum computer can be built from only two gates, namely, a one-bit unitary gate and a two-bit conditional quantum phase gate. Such gates have been discussed and implemented in many systems including trapped ions [2], cavity quantum electrodynamics (QED) [3,4], and liquid state nuclear magnetic resonance [5].

In many schemes to implement quantum logic gates, the two qubits are represented by two separate systems. For example, the internal states of an atom represent one qubit and the quantum state of the field inside the cavity represent the other in the recent cavity QED implementation of a quantum phase gate [4]. However, experimental realization of typical quantum algorithms may require the two qubits to be treated on equal footing. In this paper we discuss a quantum phase gate based on cavity QED in which the two qubits are represented by two different modes of the radiation field inside the cavity and the gate is implemented by passing a three-level atom through the cavity. Cavity QED with long-lived Rydberg states and high-Q cavities thus provides a promising tool for creating entanglement and superpositions necessary for the implementation of quantum computing algorithms.

II. QUANTUM PHASE GATE

The transformation for a two-bit quantum phase gate is given by $Q_\eta|\alpha_1,\beta_2\rangle=\exp(i\eta\delta_{\alpha_1,1}\delta_{\beta_2,1})|\alpha_1,\beta_2\rangle$, where $|\alpha_1\rangle$ and $|\beta_2\rangle$ stand for the basis states $|0\rangle$ or $|1\rangle$ of the qubits 1 and 2, respectively. Thus the quantum phase gate introduces a phase η only when both the qubits in the input states are 1. A representation of the quantum phase gate is given by the operator

$$Q_\eta=|0_1,0_2\rangle\langle 0_1,0_2|+|0_1,1_2\rangle\langle 0_1,1_2| +|1_1,0_2\rangle\langle 1_1,0_2|+e^{i\eta}|1_1,1_2\rangle\langle 1_1,1_2|, \tag{1}$$

and since $|0\rangle\langle 0|=(1+\sigma_z)/2$ and $|1\rangle\langle 1|=(1-\sigma_z)/2$, Eq. (1) has the matrix representation

$$Q_\eta=\mathbf{1}_1\mathbf{1}_2-\frac{1}{4}(1-e^{i\eta})(\mathbf{1}_1\mathbf{1}_2-\mathbf{1}_1\sigma_{z2}-\sigma_{z1}\mathbf{1}_2+\sigma_{z1}\sigma_{z2}). \tag{2}$$

Here we discuss a quantum phase gate with $\eta=\pi$.

The schematics of the quantum phase gate is shown in Fig. 1. The cavity has resonant frequencies at ν_1 and ν_2, and the photon number states $|0\rangle$ and $|1\rangle$ represent logic 0 and 1, respectively. Thus the possible cavity field states are $|0_1,0_2\rangle$, $|0_1,1_2\rangle$, $|1_1,0_2\rangle$, and $|1_1,1_2\rangle$. We consider a three-level atom in cascade configuration such that the upper two levels are resonant with the cavity mode 2, i.e., $\omega_{ab}=\nu_2$, and the lower levels are detuned by an amount Δ from the cavity mode 1, i.e., $\omega_{bc}=\nu_1+\Delta$. A quantum phase gate with a π phase shift is implemented if the atom in its ground state $|c\rangle$ passes through the cavity such that (1) the detuning Δ is equal to g_2, and (2) the interaction time τ of the atom with the cavity is such that $g_1\tau=\sqrt{2}\pi$. Here g_i $(i=1,2)$ are the vacuum Rabi frequencies associated with the interaction of the cavity modes with the respective atomic states. Under these conditions, the atomic state is decoupled from the photon states both before and after the interaction and it remains in the ground state $|c\rangle$. The cavity states also remain unaffected for the initial states $|0_1,0_2\rangle$, $|0_1,1_2\rangle$, and $|1_1,0_2\rangle$, and acquire a π phase shift only for the state $|1_1,1_2\rangle$.

In the following we discuss this phase gate, first in a dressed state picture that brings out the essential physics rather clearly. We then present the exact treatment within the dipole approximation and discuss the errors in the implementation.

First we note that the atom (which is initially in the ground state $|c\rangle$) will remain completely decoupled with the cavity field if there is no photon in the mode 1, i.e., when the cavity field states are $|0_1,0_2\rangle$ and $|0_1,1_2\rangle$. Also in the case when there is one photon in mode 1 and no photon in mode 2 ($|1_1,0_2\rangle$), the atom will again remain almost decoupled from the cavity field states if the detuning Δ is sufficiently large. The interesting situation is, however, when there is one

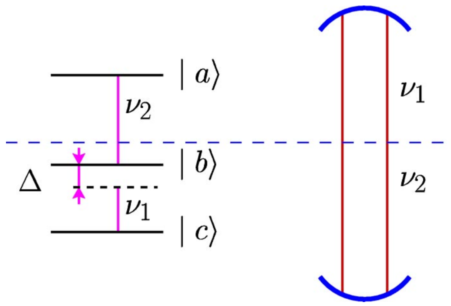

FIG. 1. The schematics of the quantum phase gate. The cavity can hold two modes of frequencies ν_1 and ν_2. The atomic levels are such that $\omega_{ab}=\nu_2$ and $\omega_{bc}=\nu_1+\Delta$.

photon each in the two modes of the cavity, i.e., the initial cavity field state is $|1_1,1_2\rangle$. We analyze this case in the following.

The effective Hamiltonian for the interaction, in the dipole and rotating-wave approximations, is

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1, \tag{3}$$

where

$$\mathcal{H}_0=\hbar\nu_1 a_1^\dagger a_1+\hbar\nu_2 a_2^\dagger a_2+\hbar\omega_{bc}|b\rangle\langle b|+\hbar\omega_{ac}|a\rangle\langle a|, \tag{4}$$

$$\mathcal{H}_1=\hbar g_1(a_1|b\rangle\langle c|+a_1^\dagger|c\rangle\langle b|)+\hbar g_2(a_2|a\rangle\langle b|+a_2^\dagger|b\rangle\langle a|), \tag{5}$$

where a_i ($a_i^\dagger$) are the photon annihilation and creation operators for the two modes. The effective Hamiltonian $\mathcal{H}_I$ in interaction picture is

$$\mathcal{H}_I\equiv e^{-i\mathcal{H}_0 t/\hbar}\mathcal{H}_1 e^{i\mathcal{H}_0 t/\hbar}=H_1+H_2,$$

where

$$\begin{aligned}H_1&=\hbar g_1(a_1|b\rangle\langle c|e^{-i\Delta t}+a_1^\dagger|c\rangle\langle b|e^{i\Delta t})\\&=\hbar g_1(|b,0,1\rangle\langle c,1,1|e^{-i\Delta t}+|c,1,1\rangle\langle b,0,1|e^{i\Delta t}),\end{aligned} \tag{6}$$

$$\begin{aligned}H_2&=\hbar g_2(a_2|a\rangle\langle b|+a_2^\dagger|b\rangle\langle a|)\\&=\hbar g_2(|a,0,0\rangle\langle b,0,1|+|b,0,1\rangle\langle a,0,0|).\end{aligned} \tag{7}$$

Here we have used the fact that, for an initial $|1_1,1_2\rangle$ state for the cavity and the state $|c\rangle$ for the atom, the only allowed states for the atom-field system are $|a,0,0\rangle$, $|b,0,1\rangle$, and $|c,1,1\rangle$.

At this point we resort to a dressed state picture and define the symmetric and antisymmetric states with respect to the field mode 2,

$$|+\rangle\equiv\frac{1}{\sqrt{2}}(|a,0,0\rangle+|b,0,1\rangle),$$

$$|-\rangle\equiv\frac{1}{\sqrt{2}}(|a,0,0\rangle-|b,0,1\rangle).$$

In terms of these states, H_2 can be rewritten as

$$H_2=\hbar g_2\{|+\rangle\langle+|-|-\rangle\langle-|\}, \tag{8}$$

with eigenvalues $\hbar g_2$ and $-\hbar g_2$. Thus the net effect of the field at frequency ν_2 is the dynamic Stark splitting. The Hamiltonian H_1 in the interaction picture of H_2 is given by

$$\begin{aligned}H_{1I}=\frac{\hbar g_1}{\sqrt{2}}\{&|+\rangle\langle c,1,1|e^{-i(\Delta+g_2)t}-|-\rangle\langle c,1,1|e^{-i(\Delta-g_2)t}\\&+|c,1,1\rangle\langle+|e^{i(\Delta+g_2)t}-|c,1,1\rangle\langle-|e^{i(\Delta-g_2)t}\}.\end{aligned} \tag{9}$$

When $g_2=\Delta$, the interaction Hamiltonian simplifies, and we obtain

$$\begin{aligned}H_{1I}=\frac{\hbar g_1}{\sqrt{2}}\{&|+\rangle\langle c,1,1|e^{-2i\Delta t}+|c,1,1\rangle\langle+|e^{2i\Delta t}-|-\rangle\\&\times\langle c,1,1|-|c,1,1\rangle\langle-|\}.\end{aligned} \tag{10}$$

For sufficiently large detuning we can ignore the oscillating terms $\exp(\pm 2i\Delta t)$, resulting in

$$H_{1I}=-\frac{\hbar g_1}{\sqrt{2}}\{|-\rangle\langle c,1,1|+|c,1,1\rangle\langle-|\}. \tag{11}$$

The effective Rabi frequency between the levels $|-\rangle$ and $|c,1,1\rangle$ is $g_1/\sqrt{2}$. Thus for an interaction time between the atom and the cavity field τ such that $g_1\tau=\sqrt{2}\pi$, we have $|c,1,1\rangle\rightarrow-|c,1,1\rangle$. This completes the description of the quantum phase gate Q_π.

In this dressed picture analysis, we ignored the possibility of excitation to the dressed state $|+\rangle$. Also, when the initial state of the cavity field is $|1,0\rangle$, there is a finite possibility of excitation to the level $|b\rangle$. This is an important source of error. In the following, we derive the exact solutions for the probability amplitudes for these two cases and discuss the amount of error.

A. Input state: $|\psi_1\rangle=|c,1,0\rangle$

For the input state $|c,1,0\rangle$, the wave vector at any time t is a superposition of $|c,1,0\rangle$ and $|b,0,0\rangle$ states, i.e., $|\psi_1\rangle=C_1(t)|c,1,0\rangle+C_2(t)|b,0,0\rangle$. Here the probability amplitudes C_1 and C_2 satisfy the following Schrödinger equations:

$$i\hbar\frac{\partial C_1}{\partial t}=\hbar g_1 C_2 e^{i\Delta t},$$

$$i\hbar\frac{\partial C_2}{\partial t}=\hbar g_1 C_1 e^{-i\Delta t}. \tag{12}$$

The initial conditions are $C_1(0)=1$ and $C_2(0)=0$. A solution of Eq. (12) subject to these conditions is

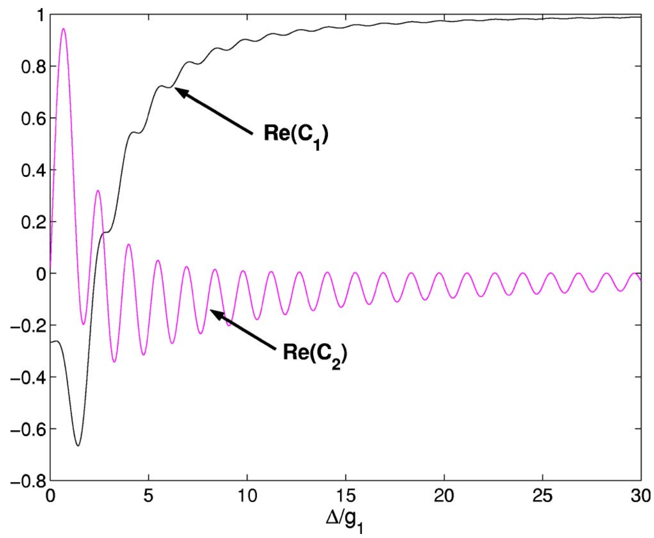

FIG. 2. Real part of probability amplitude vs normalized detuning Δ/g_1 for $g_1 t = \sqrt{2}\pi$. Dashed line is $\mathrm{Re}(C_1)$ and dotted line is $\mathrm{Re}(C_2)$.

$$C_1(t) = \frac{1}{2} e^{i\Delta t/2} \left\{ \left(1 - \frac{\Delta}{\Omega}\right) e^{i\Omega t/2} + \left(1 + \frac{\Delta}{\Omega}\right) e^{-i\Omega t/2} \right\},$$

$$C_2(t) = -\frac{g_1}{\Omega} e^{-i\Delta t/2} (e^{i\Omega t/2} - e^{-i\Omega t/2}), \tag{13}$$

where

$$\Omega \equiv \sqrt{\Delta^2 + 4g_1^2}. \tag{14}$$

In an ideal situation to implement Q_π, $C_1(t) = 1$ and $C_2(t) = 0$ may be more appropriate when $g_1 t = \sqrt{\pi}$. In Fig. 2 we plot the real parts of $C_1(t)$ and $C_2(t)$ as a function of the normalized detuning Δ/g_1. A condition for achieving the desired behavior is $\Delta/g_1 \gg 1$. For intermediate value of Δ/g_1, the state will return to its initial state $|c,1,0\rangle$ except the undesired phase factor. This additional phase factor results in an error in the implementation of quantum phase gate Q_π.

B. Initial state: $|\psi_2\rangle = |c,1,1\rangle$

For the initial state $|c,1,1\rangle$ for the atom-field system, the wave vector at any time is of the form $|\psi_2(t)\rangle = D_1(t)|c,1,1\rangle + D_2(t)|b,0,1\rangle + D_3(t)|a,0,0\rangle$. The corresponding equations for the amplitudes $D_1(t)$, $D_2(t)$, and $D_3(t)$ are

$$i\hbar \frac{\partial D_1}{\partial t} = \hbar g_1 D_2 e^{i\Delta t}, \tag{15}$$

$$i\hbar \frac{\partial D_2}{\partial t} = \hbar g_1 D_1 e^{-i\Delta t} + \hbar g_2 D_3, \tag{16}$$

$$i\hbar \frac{\partial D_3}{\partial t} = \hbar g_2 D_2. \tag{17}$$

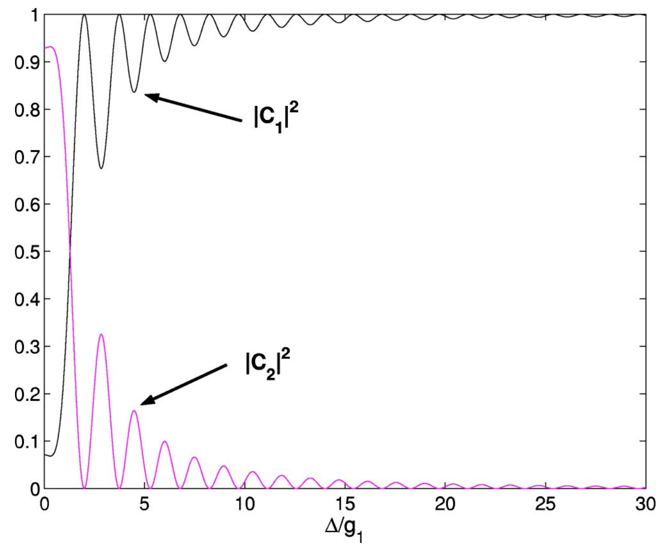

FIG. 3. Probability vs normalized detuning Δ/g_1 for $g_1 t = \sqrt{2}\pi$. Dashed line is $|C_1|^2$ and dotted line is $|C_2|^2$.

The initial conditions are $D_1(0) = 1$ and $D_2(0) = D_3(0) = 0$ (Fig. 3). We can get an exact analytical solution for these equations. These are given in the Appendix. We plot the real parts of the amplitudes in Fig. 4 and the probabilities for being in levels $|a\rangle$, $|b\rangle$, and $|c\rangle$ in Fig. 5. So, if $\Delta/g_1 \gg 5$, $D_1(t)$ will be almost close to -1, or the condition for Q_π is realized.

C. Implementation

The implementation of the quantum phase gate requires the passage of the three-level atom through a bimodal cavity. There are a number of sources of error. The atomic and cavity lifetimes should be larger than the interaction time of the atoms with the cavity fields. The photon numbers in the two

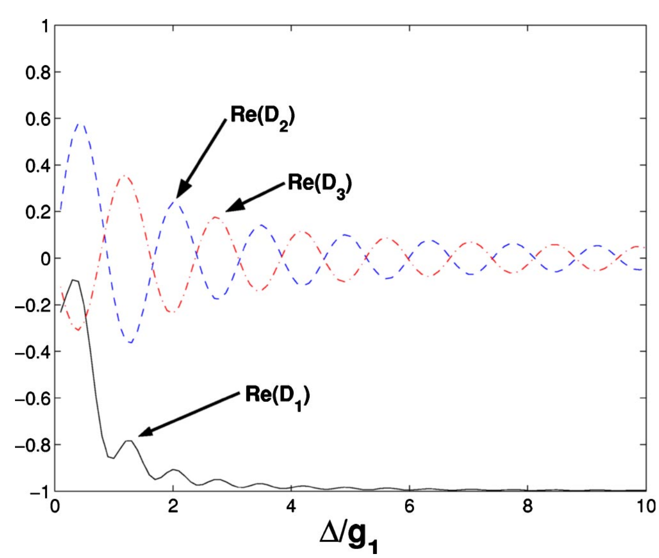

FIG. 4. Real part of probability amplitudes vs normalized detuning Δ/g_1 with the conditions $g_1 \tau = \sqrt{2}\pi$ and $g_2 = \Delta$. Solid line is $\mathrm{Re}(D_1)$, dashed line $\mathrm{Re}(D_2)$, and dotted line $\mathrm{Re}(D_3)$.

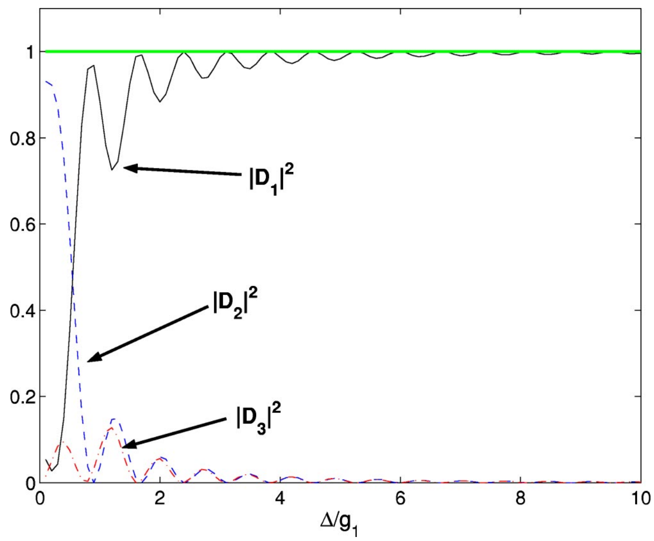

FIG. 5. Probability vs normalized detuning Δ/g_1 with the conditions $g_1\tau=\sqrt{2}\pi$ and $g_2=\Delta$. Solid line is $|D_1|^2$, dashed line $|D_2|^2$, and dotted line $|D_3|^2$.

cavity modes should also remain unaltered during the interaction with the atom. As seen above, this requires sufficiently large detuning Δ.

III. ONE-BIT UNITARY GATE

For a present case of two qubits represented by photons in the two modes of the cavity field, the one-bit unitary gate can be implemented by passing a resonant two-level atom through the cavity (see Fig. 6). In addition, a short pulse of classical field is applied midway during the passage through the cavity. Thus there are three regions for the interaction of the atom inside the cavity.

In the first region, the atom initially in the ground state $|b\rangle$ interacts with the cavity quantized field via an interaction picture Hamiltonian

$$\mathcal{H}=\hbar g(a|a\rangle\langle b|+a^\dagger|b\rangle\langle a|). \tag{18}$$

The associated unitary operator [6] is

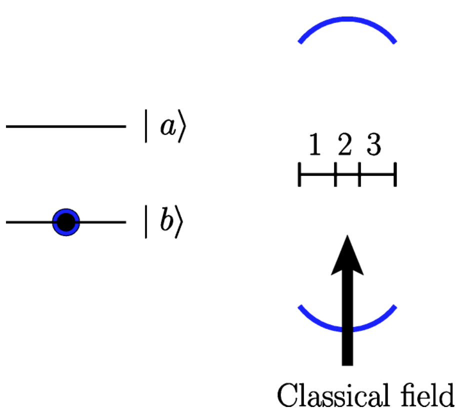

FIG. 6. The schematics of the one-bit unitary gate. The atom interacts with the resonant quantized field in regions 1 and 3 and interacts with the classical field in region 2.

$$U(t)=\cos(gt\sqrt{aa^\dagger})|a\rangle\langle a|+\cos(gt\sqrt{a^\dagger a})|b\rangle\langle b| - i\frac{\sin(gt\sqrt{aa^\dagger})}{\sqrt{aa^\dagger}}a|a\rangle\langle b|-ia^\dagger\frac{\sin(gt\sqrt{aa^\dagger})}{\sqrt{aa^\dagger}}|b\rangle\langle a|. \tag{19}$$

For interaction time $\tau=\pi/2g$, the atom-field state $|b\rangle|0\rangle$ remains unaffected; however, the state $|b\rangle|1\rangle$ evolves to $-i|a\rangle|0\rangle$. The additional phase factor $-i$ in the evolution of the state $|b\rangle|1\rangle$ results in the one-bit unitary operator $U^i_{\theta,\phi}$.

In the second region, a short classical pulse with amplitude $\mathcal{E}$ is applied, which prepares the atom in the coherent superposition:

$$|a\rangle\rightarrow\{\cos\theta|a\rangle+ie^{-i\phi}\sin\theta|b\rangle\},$$

$$|b\rangle\rightarrow\{ie^{i\phi}\sin\theta|a\rangle+\cos\theta|b\rangle\}. \tag{20}$$

Here $\theta=\Omega t/2$, where t is the interaction time, $\Omega=|p|\mathcal{E}/\hbar$ is the Rabi frequency, and ϕ is the phase of the dipole moment $p=|p|e^{i\phi}=e\langle a|x|b\rangle$. The cavity field remains in $|0\rangle$ state.

In the third region, the atom again interacts with the cavity field for the duration $\tau=\pi/2g$ and the end result is that the atom exits the cavity in state $|b\rangle$ and the cavity field is transformed according to the initial cavity states $|0\rangle$ and $|1\rangle$ as follows:

$$|0\rangle\rightarrow\{\cos\theta|0\rangle+e^{i\phi}\sin\theta|1\rangle)\}$$

$$|1\rangle\rightarrow\{e^{-i\phi}\sin\theta|0\rangle-\cos\theta|1\rangle)\}. \tag{21}$$

Thus the implemented one-bit unitary operator $U^i_{\theta,\phi}$ is

$$U^i_{\theta,\phi}=\begin{pmatrix}\cos\theta & e^{i\phi}\sin\theta\\ e^{-i\phi}\sin\theta & -\cos\theta\end{pmatrix} =\cos\theta\sigma_z+\cos\phi\sin\theta\sigma_x-\sin\phi\sin\theta\sigma_y. \tag{22}$$

The unitary operator $U^i_{\theta,\phi}$ does not satisfy the addition property for rotation even for $\phi=0$, i.e.,

$$U^i_{\theta_1+\theta_2,0}\neq U^i_{\theta_1,0}U^i_{\theta_2,0}. \tag{23}$$

As a special case, if we choose $\theta=\pi/4$ and $\phi=0$, the states $|0\rangle$ and $|1\rangle$ are mixed according to

$$|0\rangle\rightarrow\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle),$$

$$|1\rangle\rightarrow\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle). \tag{24}$$

IV. APPLICATIONS

A. Grover's algorithm

Grover's algorithm is a quantum algorithm to find the specified item from an unsorted data that contains N items. The estimated number of iteration is of the order of $O(\sqrt{N})$

and the search is accomplished with $n = \log_2 N$ qubits [7–9]. Recently, some cavity QED based implementation of Grover's algorithm have been proposed. In Refs. [10,11], the implementation scheme uses a quantum phase gate based on dispersive atomic coupling [4]. In another scheme [12], the interaction of two atoms interacting nonresonantly with the cavity field [13,14] is used. Here we consider a scheme for the implementation of Grover's algorithm with two qubits based on one- and two-bit gates discussed above.

The implementation of Grover's algorithm requires three steps [7]. The first step is to prepare the quantum system with equal amplitude state for all possible $N = 2^n$ states. The iterated step has two parts. One is to change the phase of the specified state by π, and the other is to apply the diffusion transform $\mathcal{D}$ to increase the probability of the specified state by *the inversion about the average* [7]

$$\mathcal{D}_{ij} = \frac{2}{N} - \delta_{ij}, \tag{25}$$

where δ_{ij} is the Kronecker delta. This step is carried out $\pi\sqrt{N}/4$ times to maximize the probability of the specified state. The final step is to measure the whole system to get the specified state.

We now discuss how the one-bit unitary gate $U^i_{\theta,\phi}$ and the two-bit quantum phase gate Q_π discussed here can be used to implement the Grover's algorithm for the simple case of two qubits with $N = 4$. It is known that the search process can be carried out only in one step with unit probability of success for this particular case [7,10].

The two qubits are initially prepared in the state $|0_1,0_2\rangle$, i.e., vacuum state for both modes inside the cavity. The one-bit unitary gate $U_{\pi/4,0}$ to each qubit yields the state with same amplitude for each state, i.e.,

$$\begin{aligned} |\psi\rangle &= U^1_{\pi/4,0} U^2_{\pi/4,0} |0_1,0_2\rangle \\ &= \frac{1}{2}(|0_1,0_2\rangle + |0_1,1_2\rangle + |1_1,0_2\rangle + |1_1,1_2\rangle). \end{aligned} \tag{26}$$

Next step is to change the sign of the phase in the specified state through the phase rotation, or applying the operator $C_{\alpha,\beta}$, which change the sign of the state $|\alpha_1,\beta_2\rangle$:

$$\begin{aligned} C_{0,0} &= \sigma_{x1}\sigma_{x2}Q_\pi = U^1_{\pi/2,0}U^2_{\pi/2,0}Q_\pi, \\ C_{0,1} &= \sigma_{x1}Q_\pi = U^1_{\pi/2,0}Q_\pi, \\ C_{1,0} &= \sigma_{x2}Q_\pi = U^2_{\pi/2,0}Q_\pi, \\ C_{1,1} &= Q_\pi. \end{aligned} \tag{27}$$

The *inversion about the average* operation can be carried out via the operator

$$\mathcal{D} = U^1_{\pi/2,0}U^2_{\pi/2,0}U^1_{\pi/4,0}U^2_{\pi/4,0}Q_\pi U^1_{\pi/4,0}U^2_{\pi/4,0}. \tag{28}$$

The circuit diagram corresponding to these operations involving the one-qubit unitary gate $U^i_{\theta,\phi}$ and the two-qubit quantum phase gate Q_η is shown in Fig. 7.

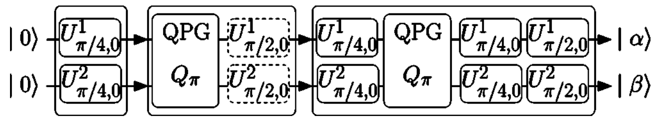

FIG. 7. Circuit diagram for the implementation of Grover's algorithm with two qubits.

In the present scheme, a high-Q cavity is initially prepared in the $|0_1,0_2\rangle$ state. The number of atoms needed in Grover's algorithm are 10–12, corresponding to one-bit and two-bit gates. The passage of these atoms with controlled interaction times will implement all the necessary steps for the Grover's algorithm. Additional two atoms may be required for the readout of the intracavity fields.

B. Measurement of Bell's basis

Another interesting application of the quantum logic gates discussed in this paper is in the measurement of Bell's basis that is crucial for quantum teleportation [15] and quantum dense coding [16].

The Bell's basis consists of the following orthogonal set of states:

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}(|0_1,1_2\rangle - |1_1,0_2\rangle), \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}(|0_1,1_2\rangle + |1_1,0_2\rangle), \\ |\psi_3\rangle &= \frac{1}{\sqrt{2}}(|0_1,0_2\rangle - |1_1,1_2\rangle), \\ |\psi_4\rangle &= \frac{1}{\sqrt{2}}(|0_1,0_2\rangle + |1_1,1_2\rangle). \end{aligned} \tag{29}$$

It can be shown that the operation

$$U^1_{3\,\pi/4,0}Q_\pi U^2_{\pi/4,0}U^1_{\pi/4,0} \tag{30}$$

on the input state yields $|0_1,0_2\rangle$, $|0_1,1_2\rangle$, $|1_1,0_2\rangle$, and $|1_1,1_2\rangle$ for the input states $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, and $|\psi_4\rangle$, respectively.

Thus, for an initial two-mode state inside the cavity, a passage of four atoms implementing three one-bit and one two-bit gate according to Eq. (30) will measure the cavity state in Bell's basis. As before, two more atoms will be required for the readout of the photon states inside the cavities.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support from Air Force Research Laboratories (Rome, New York), DARPA-QuIST, TAMU Telecommunication and Informatics Task Force (TITF) initiative, the Office of Naval Research, and the Welch Foundation.

APPENDIX: SOLUTION OF EQS. (15)–(17)

On eliminating D_1 and D_3 in Eqs. (15)–(17) we obtain the following third-order differential equation for D_2:

$$\frac{\partial^3 D_2}{\partial t^3}+i\Delta\frac{\partial^2 D_2}{\partial t^2}+(g_1^2+g_2^2)\frac{\partial D_2}{\partial t}+i\Delta g_2^2 D_2=0. \quad \text{(A1)}$$

The initial conditions for D_2 (at $t=0$) are

$$D_2=0, \quad \frac{\partial D_2}{\partial t}=-ig_1, \quad \frac{\partial^2 D_2}{\partial t^2}=-g_1\Delta. \quad \text{(A2)}$$

We consider a solution of the form

$$D_2=Ae^{\omega_1 t}+Be^{\omega_2 t}+Ce^{\omega_3 t},$$

where ω_1, ω_2, and ω_3 are the roots of the third-order polynomial

$$z^3+i\Delta z^2+(g_1^2+g_2^2)z+ig_2^2\Delta=0.$$

The coefficients A, B, and C satisfy the following linear equation:

$$\begin{pmatrix} 1 & 1 & 1 \\ \omega_1 & \omega_2 & \omega_3 \\ \omega_1^2 & \omega_2^2 & \omega_3^2 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix}=\begin{pmatrix} 0 \\ -ig_1 \\ -g_1\Delta \end{pmatrix}, \quad \text{(A3)}$$

yielding

$$A=\frac{\Delta-i(\omega_2+\omega_3)}{(\omega_1-\omega_2)(\omega_3-\omega_1)}g_1,$$

$$B=\frac{\Delta-i(\omega_3+\omega_1)}{(\omega_2-\omega_3)(\omega_1-\omega_2)}g_1,$$

$$C=\frac{\Delta-i(\omega_1+\omega_2)}{(\omega_3-\omega_1)(\omega_2-\omega_3)}g_1.$$

We can calculate D_1 and D_3 from D_2 by integrating Eqs. (17) and (19), i.e.,

$$\begin{aligned} D_1&=1-ig_1\int_0^t D_2 e^{i\Delta t}dt \\ &=-\frac{ig_1 A}{\omega_1+i\Delta}e^{(\omega_1+i\Delta)t}-\frac{ig_1 B}{\omega_2+i\Delta}e^{(\omega_2+i\Delta)t} \\ &\quad -\frac{ig_1 C}{\omega_3+i\Delta}e^{(\omega_3+i\Delta)t} \end{aligned} \quad \text{(A4)}$$

and

$$D_3=1-ig_1\int_0^t D_2 dt=-\frac{ig_1 A}{\omega_1}e^{\omega_1 t}-\frac{ig_1 B}{\omega_2}e^{\omega_2 t}-\frac{ig_1 C}{\omega_3}e^{\omega_3}. \quad \text{(A5)}$$

[1] See, for example, M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge, London, 2000).
[2] J.I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995); C. Monroe, D.M. Meekhof, B.E. King, W.M. Itano, and D.J. Wineland, *ibid.* **75**, 4714 (1995); D. Kielpinski, C. Monroe, and D.J. Wineland, Nature (London) **417**, 709 (2002).
[3] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, and H.J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).
[4] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **83**, 5166 (1999).
[5] N.A. Gershenfeld and I.L. Chuang, Science **275**, 350 (1997); J.A. Jones, Prog. Nucl. Magn. Reson. Spectrosc. **38**, 325 (2001).
[6] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge, London, 1997).
[7] L.K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
[8] E. Farhi and S. Gutmann, Phys. Rev. A **57**, 2403 (1998).
[9] S. Lloyd, Phys. Rev. A **61**, 010301 (2000).
[10] M.O. Scully and M.S. Zubairy, Proc. Natl. Acad. Sci. U.S.A. **98**, 9490 (2001).
[11] M.O. Scully and M.S. Zubairy, Phys. Rev. A **65**, 052324 (2002).
[12] F. Yamaguchi, P. Milman, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A **66**, 010302 (2002).
[13] S.-B. Zheng and G.-C. Guo, Phys. Rev. Lett. **85**, 2392 (2000).
[14] A. Sorensen and K. Molmer, Phys. Rev. A **62**, 022311 (2000).
[15] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[16] C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).