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⁶This value was estimated in Ref. (2) and confirmed by recent viscosity measurements in MBBA, cf. Ref. 9.

⁷E. Dubois-Violette, thesis, (Orsay, 1971) (unpublished).

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Temperature Dependence of the Scott Thermomagnetic Torque in N_2 , CH_4 , and HD^{\dagger}

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The temperature dependence of the Scott thermomagnetic torque has been measured in N₂, CH₄, and HD. Measurements were made from a temperature of 75 up to 300 °K, from a pressure of 0.003 up to 1 Torr, and from a magnetic field of a few oersted up to 440 Oe. In both N₂ and CH₄, the maximum torque measured at each temperature was found to increase when the temperature was increased from 75 to 300 °K. In HD this maximum torque increased when the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to about 170 °K but then decreased as the temperature was increased from 75 up to 300 °K. When the pressure is large enough, the Scott torque multiplied by the pressure plotted as a function of H/P reaches a limiting curve which has a maximum value at a field-to-pressure ratio $(H/P)_{max}$. The temperature dependence of $(H/P)_{max}$ for N₂ and CH₄ has been measured, and from these data the value for the optimum ratio of precession frequency to collision frequency $(\nu_L/\nu_c)_{opt}$ determined. The value of $(\nu_L/\nu$

I. INTRODUCTION

It has been known for sometime that the transport coefficients of all polyatomic gases change when they are placed in an external field^{1,2} (Senftleben-Beenakker effects). In this study the temperature dependence of the Scott torque, ³ a magnetothermalinduced gas-torgue effect, has been measured in N_2 , CH_4 , and HD, in order to obtain basic information and for comparison with theory. All of the Senftleben-Beenakker measurements can be explained qualitatively in terms of a model in which a rotating diatomic molecule is considered as a disk with a magnetic moment in the direction of the axis of rotation.^{4,5} Each disk clearly has a lower probability of a collision when moving in its own plane than when moving perpendicular to it; equivalently, a disk has a lower collision probability when the velocity and the rotational angular momentum vectors are perpendicular, than when they are parallel. When a thermal gradient is established in the gas the "hotter" molecules tend to move toward the colder region and the "colder" molecules toward the hotter region. In this counterflow the nonspherical molecules will tend to align because of the unequal collision probabilities for different orientations mentioned above. This flow polarization will increase the thermal conductivity λ and the shear viscosity η over the value of these quantities for a gas of spherical molecules with the same average cross section. The application of a magnetic field causes the molecules to precess, which diminishes the polarization, and this reduces the thermal conductivity and shear viscosity. The amount that the polarization is reduced and thus the degree that η and λ are changed depends on the number of precessions between two collisions or the ratio of Larmor precession frequency to collision frequency ν_L/ν_c . The effects saturate when the Larmor precession frequency reaches a value much larger than the collision frequency. Since ν_L is proportional to $g_J H$ and ν_c is proportional to P, we have

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$$\frac{\nu_L}{\nu_c} \propto g_J \frac{H}{P} \quad , \tag{1}$$

where g_J is the molecular rotational g factor. The Senftleben-Beenakker data, therefore, are plotted as a function of H/P, and we have likewise done this in Fig. 1.

A consideration of the flow polarization is also helpful for understanding the Scott torque. In the geometry of the Scott experiment, there is a nonuniform thermal gradient normal to the cylinder, and flow polarization is established in this direc-

tion. The application of a magnetic field reduces the magnitude of this polarization and also rotates the direction of the polarization away from the direction of the thermal gradient. At fields too small to rotate the polarization and at fields large enough to decrease the polarization the torque is small. The torque has a maximum at a value of the field where ν_L/ν_c has an optimum value.

Theoretical interest in the magnetic field effects has grown steadily during the past ten to fifteen years. The Boltzmann equation has been generalized to include internal degrees of freedom in classical systems, and the effect of the field on the thermal conductivity of O_2 calculated.⁶ This classical treatment has been extended to treat diatomic diamagnetic molecules (which have rotational magnetic moments of the order of a nuclear moment), and expressions for the field effect on the shear viscosity and thermal conductivity obtained.⁷ A generalization of the Boltzmann equation for quantum mechanical systems⁸ has been obtained and the solution applied to diffusion, ⁹ thermal conductivity, and viscosity.¹⁰

The Scott torgue has been treated using the quantum-mechanical equation mentioned above, where a term involving $\nabla \nabla T$ is introduced into the distribution function, ¹¹ and the results included the prediction that the torque multiplied by the pressure would be a universal function of H/P. The experimental data¹² when plotted as τP vs H/Papproach an upper "limiting curve" as the pressure is increased, which is in excellent agreement with the theory. Recently this theoretical work has been extended to include gases with paramagnetic molecules which possess a distribution of magnetic moments and comparison with measurements in NO is quite good.¹³ For complete details of the theoretical work on the magnetic effects on transport properties see the review paper by Beenakker and McCourt.²

We report in this paper new measurements on the temperature dependence of the Scott thermomagnetic torque in the three gases nitrogen, methane, and deuterium hydride. Temperature changes affect the collision frequency and the occupation number of the molecule's rotational energy levels. Nitrogen was studied and is reported here because extensive measurements at room temperature³ and initial temperature-dependence $measurements^{14}$ have been made on this gas. Deuterium hydride has been studied because the rotational energy levels are much more widely spaced than those of the other two gases, and the Scott torque is unusual in HD.¹⁵ The torque measured in HD has two maxima when plotted as a function of the magnetic field H or the field-to-pressure ratio H/P. There is disagreement over the cause of the two

peaks. It has been suggested that the torque peak that occurs at the lower field can be attributed to inelastic collisions in the gas¹⁵ (collisions that result in exhange between translational and rotational energy) or to the coupling between rotational and nuclear magnetic moments at low fields.¹⁶ The torque peak that occurs at the higher field is believed to be the normal Scott peak.

II. EXPERIMENTAL

The apparatus used in these measurements has been described in the literature. 17,18 An extremely sensitive torsion-pendulum system is used which can measure torques as small as 6.4×10^{-7} dyn cm. The system consists of an aluminum test cylinder 2. 2 cm in diameter and 10 cm long which is suspended by a quartz rod 0.076 cm in diameter and 100 cm long. Close to the upper end of the pendulum the quartz rod has been drawn into a thin fiber about 10 cm long. Attached to the rod just below the fiber is a 10-mm-diam front-faced mirror which reflects a light beam to a scale as in a sensitive galvanometer setup. The torsion constant of the fiber is 5.13×10^{-2} dyn cm/rad, and the optical path from scale to mirror is 8.28 m.

The test cylinder is surrounded (with a gap of 2.05 cm) by a copper cylinder which confines the gas to the annular region and furnishes the form on which the constantan heater is wound. The heater heats the copper cylinder and thus establishes the thermal difference of typically $\Delta T = 5$ °C at a given temperature. A thermocouple placed on a dummy cylinder above and one on a dummy cylinder below the test cylinder, and two thermocouples on the outer copper cylinder, allow the determination of thermal gradients and temperatures.

Axial fields up to 440 Oe are furnished by a pair of coils and a solenoid mounted around the pendulum system. The vertical component of the Earth's magnetic field is canceled in the vicinity of the cylinder.

Data are obtained in the following way. The system is cooled to the temperature of liquid nitrogen and the heater turned on to establish a temperature difference between the inner and outer cylinders. At this time the temperature of the apparatus starts drifting slowly upward at about 0.6 °C per minute. The torsion pendulum continuously oscillates with an amplitude of approximately 1° , and thus the light spot moves back and forth about 15 cm on the galvanometer scale. With the magnetic field off. the end points of the travel of the light spot on the scale are recorded and the center of oscillation calculated. The magnetic field is then set at a specific value, the end points on the scale again recorded, and the new center of oscillation calculated. The magnetic-field-induced torque acting on the cylinder of the torsion pendulum causes the

center of oscillation to shift. The value of this torque on the cylinder can be calculated from the difference between the center of oscillation with field off and field on, the known torsion constant, and optical-path length. After the torque is obtained the field is turned off, and the zero-field center of oscillation obtained again. The field is set to a new value and the procedure repeated. This procedure is continued for 6-8 different fields; then there is a short wait for the apparatus to reach the temperature for the next set of data points.

Nitrogen gas used in this experiment was purchased from Air Products & Chemicals of Allentown, Pa., and had a stated purity of 99.9%. The methane gas was purchased from the Matheson Company of Rutherford, N.J., and had a stated purity of at least 99.99%. The deuterium hydride was purchased from Merck Sharp and Dohme of Canada, Kirkland, Canada, and had a stated purity of 98%. The N₂ and HD gases were sent through a cold trap at 77 °K to remove moisture.

III. RESULTS

Extensive measurements of the temperature dependence of the Scott thermomagnetic torque have been made in the three gases nitrogen, methane, and deuterium hydride from a temperature of 75 °K up to room temperature and in magnetic fields of a few oersted up to 440 Oe. Typical data for nitrogen gas at T = 300 °K are shown in Fig. 1, where the torque, normalized to a temperature difference of 1 °K between the inner and outer cylinder, is plotted as a function of the magnetic field H divided by pressure P. These data were obtained for the 10 pressures 0.013, 0.018, 0.021, 0.026. 0.037, 0.047. 0.068, 0.085, 0.109, and 0.125 Torr; however, for clarity only data for 6 of these pressures are plotted in Fig. 1.

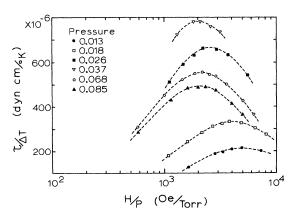


FIG. 1. Scott thermomagnetic torque in N₂ normalized to $\Delta T = 1$ °K as a function of the ratio of the magnetic field to pressure H/P (log scale).

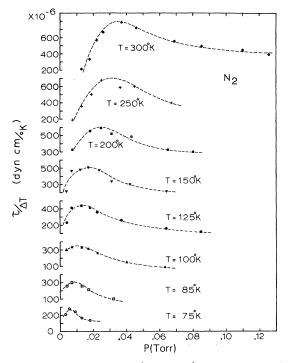


FIG. 2. Maxima of the $\tau/\Delta T$ -vs-H/P curves for N₂ (similar to those in Fig. 1) plotted as a function of pressure at temperatures T=300, 250, 200, 150, 125, 100, 85, and 75° K.

For each pressure as H/P is increased the normalized torque passes through a maximum. This maximum value of the torque and the value of H/Pfor maximum torque depend on the pressure of the gas. The pressure dependence of the maxima of the curves in Fig. 1 is shown as the top curve in Fig. 2. This maximum torque as a function of pressure increases as the pressure is increased, reaching a maximum and then decreasing as the pressure is increased further. Thus the maximum of all the maxima for the normalized torque at T = 300 °K occurs at $P_{max} = 0.035$ Torr and has a value of $(\tau/\Delta T)_m = 800 \times 10^{-6}$ dyn cm °K. Similar data are shown in Fig. 2 for temperatures of 250, 200, 150, 125, 100, 85, and 75 $^{\circ}$ K. The pressure for the maximum of the maxima and the magnitude of the normalized torque at that pressure (the normalized torque maximum) become smaller as the temperature is lowered.

In Fig. 3 the normalized torque maxima from Fig. 2 are plotted as a function of temperature on a semilog graph and are observed to increase from a low value of $(\tau/\Delta T)_m = 270 \times 10^{-6} \text{ dyn cm/}^{\circ} \text{K}$ at T= 75 °K to a value of $(\tau/\Delta T)_m = 800 \times 10^{-6} \text{ dyn cm/}^{\circ} \text{K}$ at T = 300 °K along a straight line.

The Scott torque, normalized to $\Delta T = 1$ °K between inner and outer cylinders when multiplied by the pressure at which the torque is measured, plotted

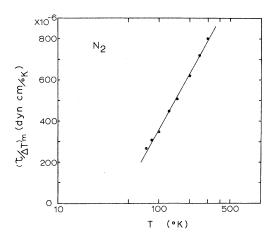


FIG. 3. Normalized torque maxima in N_2 (maxima of curves in Fig. 2) as a function of temperature (log scale).

as a function of field to pressure H/P approaches an upper limiting curve as the pressure is increased.^{11,12} The maximum value of this limiting curve depends on the temperature and of course the gas. The temperature dependence of the maxima of the limiting curves for N₂ is shown in Fig. 4, and these data increase along a smooth curve from the smallest value of 2. 1×10^{-6} Torr dyn cm/ °K at T = 75 °K to a value of 50×10^{-6} Torr dyn cm/ °K at T = 300 °K.

The maximum of the limiting curve occurs at a value of the field-to-pressure ratio $(H/P)_{max}$ which is also characteristic of the gas that is being studied. The temperature dependence of $(H/P)_{max}$ for nitrogen is shown in Fig. 5, where the value of $(H/P)_{max}$ is plotted as a function of the reciprocal of the square root of the temperature. This linear dependence of $T^{-1/2}$ was expected from the

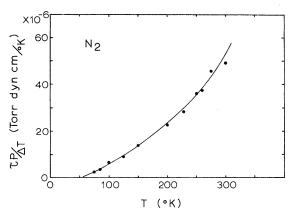


FIG. 4. Maxima of the limiting curve for N_2 (the curve formed by plotting $\tau P/\Delta T \text{ vs } H/P$ at sufficiently high pressure where Knudsen effects are unimportant) as a function of temperature.

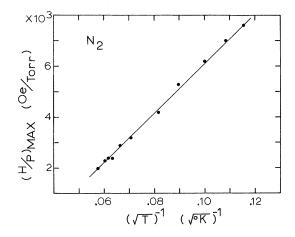


FIG. 5. Ratio of magnetic field to pressure $(H/P)_{max}$, where the limiting curve has the maximum value plotted as a function of the reciprocal of the square root of the temperature. Measurements are in N₂ gas.

consideration of the collision frequency and the precession frequency and will be discussed in Sec. IV. The value of $(H/P)_{max}$ for nitrogen decreases from 7600 Oe/Torr at 75 °K to 2000 Oe/Torr at 300 °K.

Measurements similar to those made in nitrogen and described above have been made in methane gas and these results are shown in Figs. 6 and 7. The maxima of the normalized torque vs H/P were plotted as a function of P and a set of curves obtained for the temperature range 80-300 °K. The maxima of these torque-vs-pressure data are shown as a function of temperature on a semilog plot in Fig. 6, and these data are observed to increase along a straight line from a value of 24×10^{-6} dyn cm/°K at T = 80 °K to a value of 83×10^{-6} dyn cm/°K at T = 300 °K. The maxima

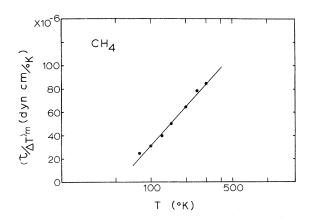


FIG. 6. Normalized torque maxima in CH_4 as a function of temperature (log scale).

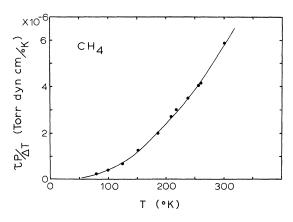


FIG. 7. Maxima of the limiting curves for CH_4 as a function of temperature.

of the limiting curves for methane are plotted as a function of temperature in Fig. 7, and a curve is drawn through the data increasing smoothly from a value of 0.2×10^{-6} Torr dyn cm/°K at T= 80 °K to a value of 5.9×10⁻⁶ Torr dyn cm/°K at T = 300 °K.

It will be recalled that the gas HD exhibits two maxima in the plot of torque vs magnetic field. Temperature-dependence data for both peaks were obtained in the same way the data for N_2 and CH_4 were obtained and these data are given on a semilog plot in Fig. 8 (triangles for low field and circles for high field). For the low-field data the torque maxima increase from a value of 93×10^{-6} dyn cm/°K at T = 90 °K along a straight line to a value of 144×10^{-6} dyn cm/°K at T = 160 °K. Above the temperature of 160 °K, the data deviates from the straight line and decreases as the temperature is increased to 300 °K. The data for the highfield peak also increase along a straight line from a value of 114×10^{-6} dyn cm/°K at 90 °K to a value of 184×10^{-6} dyn cm/°K at 180 °K. Above this temperature of 180 °K, the torque deviates from the straight line and decreases as the temperature is increased to 300 °K.

IV. DISCUSSION

As was pointed out in Sec. III if the pressure is large enough the torque-times-pressure data reach a limiting curve which has a maximum value of $(H/P)_{max}$. Thus at each temperature there is an optimum value of the field-to-pressure ratio which is temperature dependent and is a linear function of the reciprocal of the square root of the temperature (see Fig. 6). A value of the ratio of the precession frequency to the collision frequency ν_L/ν_c for nitrogen can be obtained from the slope of the line through the data points in Fig. 6. The precession frequency for a molecule with a rotational magnetic moment $\mu_{\text{mot}} = g_J \mu_N J$ is

$$\nu_L = g_J \mu_N H/h , \qquad (2)$$

where g_J is the molecular rotational g factor, μ_N is the nuclear magneton, h is Planck's constant, and H is the magnitude of the magnetic field. The collision frequency is

$$\nu_{\sigma} = \frac{2.45\sigma}{(mk)^{1/2}} \frac{P}{T^{1/2}} , \qquad (3)$$

where σ is the collision cross section, *m* is the mass of the molecule, *k* is the Boltzmann constant, *P* is the pressure, and *T* is the temperature. Therefore, the ratio of ν_L/ν_c is

$$\frac{\nu_L}{\nu_c} = \frac{(mk)^{1/2} g_J \mu_N}{2.45 \sigma h} \frac{H}{P} T^{1/2} .$$
 (4)

For each temperature T there is a value of field to pressure $(H/P)_{max}$ for which the torque is maximum and thus there is an optimum frequency ratio $(\nu_L/\nu_c)_{opt}$. Equation (4) then becomes

$$\left(\frac{H}{P}\right)_{\text{max}} = \left[\frac{2.45\sigma h}{(mk)^{1/2}g_J\mu_N} \left(\frac{\nu_L}{\nu_c}\right)_{\text{opt}}\right] T^{-1/2} .$$
(5)

Equating the square brackets of Eq. (5) to the slope of the line in Fig. 6 gives a value $(\nu_L/\nu_c)_{opt}$ = 0.113. Similar treatment of data for CH_4 gives a value $(\nu_L/\nu_c)_{opt} = 0.128$. These data thus indicate that the conditions for maximum torque are those for which the molecules precess on the average approximately $\frac{1}{8}$ of a turn between collisions. Since $(H/P)_{max}$ is temperature dependent, it should be noted that previously reported "roomtemperature" values were for temperatures from T = 300 °K up to T = 350 °K. For example, the room-temperature value of $(H/P)_{max}$ for nitrogen has been reported as $(H/P)_{max} = 1750 \text{ Oe}/\text{Torr}$, which was for a temperature of the inner cylinder of about 330 °K. This is in good agreement with what would be expected from the data of Fig. 5.

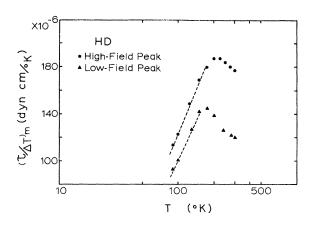


FIG. 8. Normal torque maxima of both peaks in HD as a function of temperature (log scale).

The temperature dependence of the normalized torque maximum $(\tau/\Delta T)_m$ for N₂ (Fig. 3) and for CH_4 (Fig. 6) is similar, and when plotted as a function of temperature on semilog paper, both sets of data (for N_2 and CH_4) form straight lines. For N_2 and CH_4 the torque increases continuously about a factor of 3 as the temperature is increased from a low value at about 80 to the high value at about 300 °K.

At the lower temperatures in HD for both the low-field peak and the high-field peak, the normalized torque maximum increases as the temperature is increased much the same as it does in N_2 and CH₄. However, at about 160-180°K the data deviate from the straight lines in Fig. 8, and the normalized torque decreases with increasing temperature. This sudden decrease is puzzling since as the temperature is increased both the rotational energy and the number of molecules in the upper ro-

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tational levels are increased. It would be interesting to extend the measurements in N₂ or CH₄ to higher temperatures to see if the decrease with increasing temperature is unique to HD, or if it is a general property of the Scott torque. These higher temperature measurements are not possible with the present apparatus.

Levi and Beenakker¹¹ predicted and experiment¹² has shown that if the pressure is high enough to avoid Knudsen effects the Scott torque times the pressure is a universal function of H/P. At present none of the theories reported predict how this function, torque times pressure, should depend on the temperature. In this paper measurements are presented for the temperature dependence of the normalized torque times pressure for both N₂ and CH₄, and now it would be very interesting and exciting to have the theoretical predictions of this temperature dependence.

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Interaction of Charged Vortex Rings with a Superfluid Helium Surface*

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Electrons carried by vortex rings toward a free surface in liquid He⁴ are found to evaporate from the surface with an energy independent of the ring energy. The energy of the electron is found to depend upon the amount of charge stored near the surface. We find also that the percentage of the ring current which crosses the surface depends on temperature, ring energy, and conditions at the surface. Finally, we hypothesize a trapped state near the surface and analyze our results in terms of the lifetime of this state.

I. ENERGY MEASUREMENTS

In 1968¹ evidence was published that negatively charged vortex rings interacting with the free liquid surface gave up their entire energy to an electron.

This work has not since been substantiated nor explained. The main difference between the work we report here and the previous work is that we surrounded the liquid surface with an insulator (Lucite), or a conductor whose potential could be