$D_{sJ}(2317)$ meson production in ultrarelativistic heavy ion collisions

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Production of $D_{sJ}(2317)$ mesons in relativistic heavy ion collisions at the BNL Relativistic Heavy Ion Collider is studied. Using the quark coalescence model, we first determine the initial number of $D_{sJ}(2317)$ mesons produced during hadronization of created quark-gluon plasma. The predicted $D_{sJ}(2317)$ abundance depends sensitively on the quark structure of the $D_{sJ}(2317)$ meson. An order-of-magnitude larger yield is obtained for a conventional two-quark than for an exotic four-quark $D_{sJ}(2317)$ meson. To include the hadronic effect on the $D_{sJ}(2317)$ meson yield requires the absorption cross sections of the $D_{sJ}(2317)$ meson by pions, ρ mesons, anti-kaons, and vector anti-kaons, which we have evaluated in a phenomenological model. Taking into consideration the absorption and production of $D_{sJ}(2317)$ mesons during the hadronic stage of heavy ion collisions via a kinetic model, we find that the final yield of $D_{sJ}(2317)$ mesons remains sensitive to its initial number produced from the quark-gluon plasma, providing thus the possibility of studying the quark structure of the $D_{sJ}(2317)$ meson and its production mechanism in relativistic heavy ion collisions.

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I. INTRODUCTION

A narrow $D_{sJ}(2317)$ meson was recently observed by the BABAR Collaboration [1] in the inclusive $D_s^+\pi^0$ invariant mass distribution from e^+e^- annihilation and confirmed by the Belle Collaboration in B meson decay [2]. This meson has the natural spin-parity $J^P = 0^+$ and a mass below that obtained from the QCD sum rule approach [3] and quark model calculations [4] for a normal two-quark state $c\bar{s}$. The $D_{sJ}(2317)$ meson has thus been considered as a possible candidate for the exotic four-quark states that were studied in the bag model [5,6], QCD sum rules [7], and the nonrelativistic potential model [8]. It is also possible that the $D_{sJ}(2317)$ meson is simply a DK molecule or atom. Determination of the $D_{sJ}(2317)$ meson width is limited by experimental resolutions to a value of less than 4.6 MeV$/c^2$ [2]. The small width of the $D_{sJ}(2317)$ meson is not surprising, as its mass is below the threshold of the DK system and can only decay into the kinematically allowed but isospin violated channel of the $D_s\pi$ state. Theoretically, the decay width of $D_{sJ} \to D_s\pi$ has been studied using the QCD sum rules, and its value varies from a few keV [9] to a few tens keV [10] depending on the assumed flavor state of four quarks or two quarks, respectively. A more phenomenological approach based on the $^3P_0$ model [11] also gives a narrow width of about a few tens keV for a normal two-quark $D_{sJ}(2317)$ meson.

Studying the mechanism for $D_{sJ}(2317)$ meson production in nuclear reactions is useful for understanding its quark structure. In Ref. [12], a coupled-channel quark model was used to study the production of a two-quark $D_{sJ}(2317)$ meson and its radial excitations in hadronic reactions. Production of $D_{sJ}(2317)$ in relativistic heavy ion collisions has also been studied [13]. It was found that for a four-quark $D_{sJ}(2317)$ meson, a much larger yield is obtained if one takes into account the diquark-diquark interactions in the produced quark-gluon plasma. Since the $D_{sJ}(2317)$ meson is not expected to survive in the quark-gluon plasma, it is more likely to be produced at hadronization of the quark-gluon plasma either statistically or via quark coalescence. Its final abundance in a heavy ion collision depends also, however, on its absorption and production probability in the subsequent hadronic matter.

In the present paper, we study $D_{sJ}(2317)$ meson production in central heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) in a kinetic model that starts from the final stage of the quark-gluon plasma, goes through a mixed phase of quark-gluon and hadronic matters, and finally undergoes the hadronic expansion. The production of $D_{sJ}(2317)$ mesons from the quark-gluon plasma is modeled by the constituent quark coalescence model, which has been shown to describe reasonably well not only the particle yields and their ratios [14] but also their transverse momentum spectra and anisotropic flows [15–17]. The predicted number of $D_{sJ}(2317)$ mesons is found to depend on its quark structure, with the two-quark state giving an order-of-magnitude larger value than the four-quark state. The $D_{sJ}(2317)$ meson can be absorbed and produced in subsequent hadronic matter via the reactions $\pi D_{sJ} \leftrightarrow K^*D(KD^*)$, $\rho D_{sJ} \leftrightarrow KD$, $\bar{K}D_{sJ} \leftrightarrow \rho D(\pi D^*)$, and $\bar{K}^*D_{sJ} \leftrightarrow \pi D$. The cross sections for these reactions are evaluated in a phenomenological hadronic model with coupling constants and form factors involving the $D_{sJ}(2317)$ meson determined from the QCD sum rules. Taking into account the hadronic effect in heavy ion collisions via a kinetic approach, we find that the final number of $D_{sJ}(2317)$ mesons remains sensitive to its initial number produced from the quark-gluon plasma. Studying $D_{sJ}(2317)$ meson

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production in heavy ion collisions thus provides the possibility of studying both its production mechanism and its quark structure.

This paper is organized as follows. In Sec. II, the dynamics of heavy ion collisions at RHIC is described. Production of the $D_{sJ}(2317)$ meson from the initial quark-gluon plasma via the quark coalescence model is discussed in Sec. III. The absorption cross sections of the $D_{sJ}(2317)$ meson by hadrons such as the pion, ρ, anti-kaon, and vector anti-kaon as well as their thermally averaged values are evaluated in Sec. IV. Solving the rate equation based on a kinetic model, the time evolution of the $D_{sJ}(2317)$ meson abundance in heavy ion collisions is presented in Sec. V. A summary is then given in Sec. VI. Details on the derivation of an approximate analytical coalescence formula for $D_{sJ}(2317)$ meson production from the quark-gluon plasma is given in Appendix A, while those on the QCD sum-rule approach to the determination of the form factor at the $D_{sJ}DK$ vertex, which is needed in calculating the $D_{sJ}(2317)$ meson absorption cross sections, is described in Appendix B.

II. HEAVY ION COLLISION DYNAMICS AT RHIC

To model the dynamics of central relativistic heavy ion collisions after the end of the quark-gluon plasma phase, we use the schematic model of Ref. [18] based on the boost invariant picture of Bjorken [19] and an accelerated transverse expansion. Instead of solving the hydrodynamic equation with certain equations of state, we take the volume of produced fire-cylinder in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, which is the collision we are interested in, to evolve with the proper time according to

$$V(\tau) = \pi[R_C + v_C(\tau - \tau_C) + a/2(\tau - \tau_C)^2]^2 \tau c, \quad (1)$$

where $R_C = 8$ fm and $\tau_C = 5$ fm/c are final transverse and longitudinal sizes of the quark-gluon plasma, while $v_C = 0.4c$ is its transverse flow velocity at this time. The total transverse energy of quarks and gluons in the midrapidity ($|y| \leqslant 0.5$) is then about 1067 GeV if quarks and gluons are taken to be massive with $m_g = 500$ MeV, $m_u = m_d \equiv m_q = 300$ MeV, and $m_s = 475$ MeV in order to take into account the nonperturbative effects of QCD near the critical temperature [20], and if the quark strangeness and baryon chemical potentials are taken to be $\mu_s = 0$ and $\mu_b = 10$ MeV, respectively, to account for strangeness neutrality in the quark-gluon plasma and the observed final antiproton to proton ratio of about 0.7 at RHIC. The fire-cylinder then goes through a mixed phase of partonic and hadronic matters at a constant temperature T_C until $\tau_H = 7.5$ fm/c when its transverse radius and flow velocity are $R_H \approx 9$ fm and $v_H \approx 0.45c$, respectively. This corresponds to a small transverse acceleration $a = 0.02\ c^2$/fm, which is chosen to reflect the small pressure near phase transition and in hadronic matter [21] as well as to obtain a lifetime for the expanding matter comparable to that from the transport model [22]. Values of other parameters of the fire-cylinder are determined from fitting the measured transverse energy $\simeq$788 GeV as well as the extracted freeze-out temperature $T_F = 125$ MeV and transverse flow velocity $\simeq 0.65c$ of midrapidity hadrons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Both the fraction of hadronic matter during the mixed phase and the time dependence of the temperature of the fire-cylinder after the mixed phase are determined by assuming that the fire-cylinder expands isentropically. It was found in Ref. [18] that the former increases approximately linearly and the latter can be parametrized as

$$T(\tau) = T_C - (T_H - T_F)\left(\frac{\tau - \tau_H}{\tau_F - \tau_H}\right)^{0.8}, \quad (2)$$

where T_H is the temperature of the hadronic matter at the end of the mixed phase and is thus the same as the critical temperature T_C for the quark-gluon plasma to hadronic matter transition. As in Ref. [18], we take $T_H = T_C = 175$ MeV. The freeze-out temperature $T_F = 125$ MeV then leads to a freeze-out time $\tau_F \approx 17.3$ fm/c.

For normal hadrons such as pions, kaons, anti-kaons, and nucleons, they are taken to be in chemical equilibrium with the baryon chemical potential $\mu_B = 30$ MeV, charge chemical potential $\mu_Q = 0$ MeV, and strangeness chemical potential $\mu_S = 10$ MeV. The nonzero strange chemical potential is needed to account for the observed $K^-/K^+ \approx 0.9$ ratio in heavy ion collisions at RHIC. Neglecting the time dependence of the chemical potentials, which have been shown to vary weakly with the temperature of an isentropically expanding matter in heavy ion collisions at RHIC [23], time evolution of the abundance of pions, kaons, anti-kaons, and nucleons has been shown in Fig. 1 of Ref. [18]. Including also the contributions from the decays of resonances, the time evolution of the abundance of these hadrons is shown in Fig. 1(a). It is seen that the total numbers of pions, kaons, and anti-kaons do not change much during the hadronic stage, while the nucleon number decreases significantly as temperature drops. Their final numbers at freeze-out are 926, 133, 113, and 47, respectively, and are close to those measured in experiments.

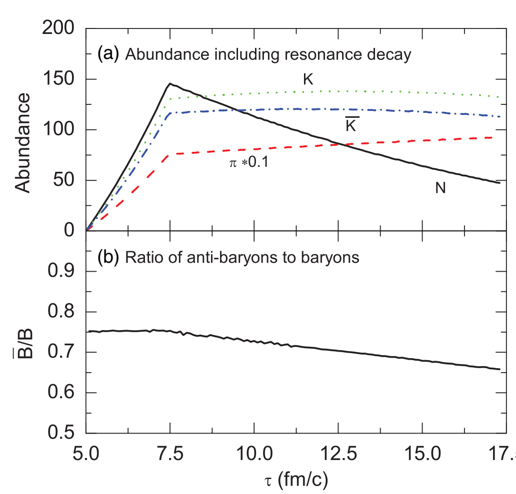

FIG. 1. (Color online) Time evolution of the abundance of pions, kaons, anti-kaons, and nucleons including contributions from decays of resonances (a) and the ratio of anti-baryon to baryon abundances (b) of mid-rapidity particles in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

In Fig. 1(b), the time evolution of the ratio of anti-baryons to baryons is shown, and it changes from an initial value of 0.75 to a final value of 0.66, which is also close to the measured value of about 0.7.

III. D_{sJ}(2317) MESON PRODUCTION FROM THE QUARK-GLUON PLASMA

A. Coalescence model

In the coalescence model, the number of $D_{sJ}(2317)$ mesons that are produced from the quark-gluon plasma is given by the product of a statistical factor $g_{D_{sJ}}$ which denotes the probability of combining $c\bar{s}$ or $c\bar{s}q\bar{q}$ quarks into a color neutral, spin 0, and isospin 0 hadronic state and depends on whether the $D_{sJ}(2317)$ meson is a two-quark or four-quark state, and the overlap of the quark phase-space distribution function $f_q(x_i, p_i)$ in the fire-cylinder with the Wigner distribution function $f^W_{D_{sJ}}$ of the $D_{sJ}(2317)$ meson. The latter corresponds to the probability of converting the above partonic state into $D_{sJ}(2317)$ and is dependent on the quark spatial wave functions in the $D_{sJ}(2317)$ meson. Explicitly, the $D_{sJ}(2317)$ number is expressed as

$$N^{\rm coal}_{D_{sJ}} = g_{D_{sJ}} \int_{\sigma_C} \prod_{i=1}^{n} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3\mathbf{p}_i}{E_i} f_q(x_i, p_i) \times f^W_{D_{sJ}}(x_1 \ldots x_n; \quad p_1 \ldots p_n), \tag{3}$$

with $n = 2$ or 4 for a two-quark or four-quark $D_{sJ}(2317)$ meson. Similar expressions have previously been used for studying the production of strange hadrons [24], charmed mesons [25], and penta-quark baryons [18] from the quark-gluon plasma formed in relativistic heavy ion collisions.

We note that the coalescence model can be viewed as the formation of bound states from interacting particles with energy mismatch balanced by other particles in the system. Neglecting such off-shell effects is reasonable if the binding energy is not large and/or if the production process is fast compared to the inverse of the energy mismatch.

Since the normalized quark wave function of the $D_{sJ}(2317)$ meson in the color-spin-isospin space can be expressed as a linear combination of all possible orthogonal flavor, color, and spin basis states, with coefficients depending on the quark model used for the $D_{sJ}(2317)$ meson, then $g_{D_{sJ}}$ is simply given by the probability of finding these quarks in any one of these color-spin-isospin basis states, i.e., $g_{D_{sJ}} = 1/3^2 \times 1/2^2 = 1/36$ or $1/3^4 \times 1/2^4 = 1/1296$ for a two-quark or four-quark $D_{sJ}(2317)$ meson, respectively.

The $d\sigma$ in Eq. (3) denotes an element of a spacelike hypersurface σ_C at hadronization [26]. In terms of the proper time $\tau = (t^2 - z^2)^{1/2}$, the longitudinal momentum-energy rapidity $y = \frac{1}{2}\ln(\frac{E+p_z}{E-p_z})$ and space-time rapidity $\eta = \frac{1}{2}\ln(\frac{t+z}{t-z})$, the polar coordinates r and ϕ in the transverse plane, the covariant volume element can be written as $p\cdot d\sigma = \tau m_T \cosh(y-\eta) r\, dr\, d\phi\, d\eta$.

For the phase-space distribution functions of quarks in the fire-cylinder, they are taken to be the same as in the description of the heavy ion collision dynamics; i.e., they are uniformly distributed in the transverse plane, and their momentum distributions are relativistic Boltzmannian in the transverse direction but uniform in rapidity along the longitudinal direction. Also, the Bjorken correlation of equal spatial η and momentum y rapidities is imposed, which is consistent with the small difference $|y - \eta| \leqslant 0.5$ seen in the transport model [22]. Explicitly, the quark momentum distribution per unit rapidity at T_C is

$$f_q(\eta, \mathbf{r}, y, \mathbf{p}_T) = g_q \delta(\eta - y) \exp\left(\frac{-\gamma(m_T - \mathbf{p}_T\cdot\beta) - \mu_q}{T_C}\right), \tag{4}$$

where $g_q = 6$ is the color-spin degeneracy of a quark, $\beta = (\mathbf{r}/R)v_C$ is the radial-dependent transverse flow velocity, $\gamma = (1-\beta^2)^{-1/2}$, and the quark chemical potential $\mu_q = \mu_b + \mu_s$. The abundances of quarks at the end of the QGP phase in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV described by the expanding fire-cylinder model of the previous section are $N_u = N_d \simeq 245$, and $N_{\bar{s}} \simeq 149$ at τ_C, if we take into account the effect of gluons by converting them into quarks according to the quark flavor composition in the quark-gluon plasma as in Ref. [15]. For charm quarks, we assume that they are in thermal equilibrium in the quark-gluon plasma, which is supported by the large elliptic flow of electrons from charmed meson decays that are observed in experiments [27,28] and transport models [29,30]. Their number, however, is presently uncertain, as it depends on whether charm quarks can be produced from the quark-gluon plasma. If we assume that the latter contribution is unimportant, then the number of charm quarks N_c produced in heavy ion collisions is simply given by the product of the charm quark number N_c^{NN} produced from an initial hard nucleon-nucleon scattering and the total number of binary collisions (about 960) in central Au+Au collisions. Using N_c^{NN} from the PYTHIA program, we obtain $N_c \sim 1.5$. The value of N_c increases to about 3 and 7 if we use the N_C^{NN} from the PHENIX [31] and STAR experiments [32], respectively. In the following study, we will use $N_c = 3$ for the calculation and discuss the sensitivity of our results to the change in the value for N_c. For the charm quark mass, it is taken to be $m_c = 1.5$ GeV.

For the Wigner distribution function of the $D_{sJ}(2317)$ meson, instead of using a function of Lorentz invariant four-dimensional relative coordinates and momenta such as in Refs. [15,33], we take it to be a function of three-dimensional relative coordinates and momenta for simplicity. Specifically, it is obtained by assuming that the wave functions of the quarks are those of a harmonic oscillator with an oscillator frequency ω. For a two-quark $D_{sJ}(2317)$ meson with $J^\pi = 0^+$, its $c\bar{s}$ quarks are in the relative p wave, and its Wigner distribution function is thus [33]

$$f^{W,\rm two}_{D_{sJ}}(x;p) = \left(\frac{16}{3}\frac{\mathbf{y}^2}{\sigma^2} - 8 + \frac{16}{3}\sigma^2\mathbf{k}^2\right)\exp\left(-\frac{\mathbf{y}^2}{\sigma^2} - \sigma^2\mathbf{k}^2\right). \tag{5}$$

In the above, we have used the usual definitions $\mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{k} = (m_s\mathbf{p}_1 - m_c\mathbf{p}_2)/(m_c + m_s)$ for the relative coordinate and momentum of the two quarks, respectively. For the width parameter σ, it can be related to the oscillator frequency ω by $\sigma = 1/(\mu\omega)^{1/2}$ with the reduced mass μ given by $\mu = m_c m_s/(m_c + m_s)$.

If the $D_{sJ}(2317)$ meson is a four-quark meson, we assume that its four quarks are all in relative s waves, which then leads to the Wigner distribution function

$$f_{D_{sJ}}^{W,\text{four}}(x;p) = 8^3 \exp\left(-\sum_{i=1}^{3}\frac{\mathbf{y}_i^2}{\sigma_i^2} - \sum_{i=1}^{3}\mathbf{k}_i^2\sigma_i^2\right), \quad (6)$$

where the relative coordinates $\mathbf{y}_i$ and momenta $\mathbf{k}_i$ are related to the quark coordinates $\mathbf{x}_i$ and momenta $\mathbf{p}_i$ by the Jacobian transformations

$$\begin{aligned}
\mathbf{y}_1 &= \frac{\mathbf{x}_1 - \mathbf{x}_2}{\sqrt{2}},\\
\mathbf{y}_2 &= \sqrt{\frac{2}{3}}\left(\frac{m_c}{m_c+m_s}\mathbf{x}_1 + \frac{m_s}{m_c+m_s}\mathbf{x}_2 - \mathbf{x}_3\right),\\
\mathbf{y}_3 &= \sqrt{\frac{3}{4}}\left(\frac{m_c}{m_c+m_s+m_q}\mathbf{x}_1 + \frac{m_s}{m_c+m_s+m_q}\mathbf{x}_2 + \frac{m_q}{m_c+m_s+m_q}\mathbf{x}_3 - \mathbf{x}_4\right),
\end{aligned} \quad (7)$$

and

$$\begin{aligned}
\mathbf{k}_1 &= \sqrt{2}\frac{m_s\mathbf{p}_1 - m_c\mathbf{p}_2}{m_c+m_s},\\
\mathbf{k}_2 &= \sqrt{\frac{3}{2}}\frac{m_q(\mathbf{p}_1+\mathbf{p}_2) - (m_c+m_s)\mathbf{p}_3}{m_c+m_s+m_q},\\
\mathbf{k}_3 &= \sqrt{\frac{4}{3}}\frac{m_q(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3) - (m_c+m_s+m_q)\mathbf{p}_4}{m_c+m_s+2m_q}.
\end{aligned} \quad (8)$$

It can be shown that the product of the Jacobians for the coordinate and momentum transformations is equal to unity.

The width parameter σ_i for the ith relative coordinate in a four-quark $D_{sJ}(2317)$ meson is again given by $\sigma_i = 1/(\mu_i\omega)^{1/2}$ with the reduced masses

$$\begin{aligned}
\mu_1 &= \frac{2m_c m_s}{m_c+m_s},\\
\mu_2 &= \frac{3}{2}\frac{m_q(m_c+m_s)}{m_c+m_s+m_q},\\
\mu_3 &= \frac{4}{3}\frac{m_q(m_c+m_s+m_q)}{m_c+m_s+2m_q}.
\end{aligned} \quad (9)$$

We note that the reduced mass μ_1 of the $c\bar{s}$ quark pair in a four-quark $D_{sJ}(2317)$ meson is a factor of 2 larger than that in a two-quark $D_{sJ}(2317)$ meson because of differences in the definitions of the relative coordinate and momentum.

B. Number of $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma

Evaluating the number of $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma requires information on the oscillator frequency ω through the width parameter σ in the $D_{sJ}(2317)$ meson Wigner distribution function, which is related to the size of $D_{sJ}(2317)$. Since the latter is not known empirically, we choose the value of the oscillator frequency to fit instead the root-mean-square charge radius of the s-wave charmed $D_s^+(c\bar{s})$ meson. Taking its Wigner distribution function similar to Eq. (6) but with only one relative coordinate and momentum, we obtain the following mean-square charge radius for the $D_s^+(c\bar{s})$ meson:

$$\begin{aligned}
\langle r_{D_s}^2\rangle_{\text{ch}} &= \frac{2}{3}\langle(\mathbf{x}_1-\mathbf{Y})^2\rangle + \frac{1}{3}\langle(\mathbf{x}_2-\mathbf{Y})^2\rangle\\
&= \frac{m_c^2+2m_s^2}{3(m_c+m_s)^2}\langle\mathbf{y}^2\rangle = \frac{m_c^2+2m_s^2}{2(m_c+m_s)^2}\sigma^2.
\end{aligned} \quad (10)$$

In the above, $\mathbf{Y} = (m_c\mathbf{x}_1 + m_s\mathbf{x}_2)/(m_c+m_s)$ is the center-of-mass coordinate of $c\bar{s}$ quarks, and we have used the relation $\langle\mathbf{y}^2\rangle = (3/2)\sigma^2$ between the mean-square distance and the width parameter for two quarks in the relative s wave as in the $D_s^+(c\bar{s})$ meson. Using the value $\langle r_{D_s}^2\rangle_{\text{ch}} \approx 0.124$ fm^2 determined from the light-front quark model [34], we find that $\sigma \approx 0.60$ fm and $\hbar\omega \approx 300$ MeV.

For a two-quark $D_{sJ}(2317)$ meson, whose quarks are in the relative p wave, the relation between the mean-square distance of the two quarks and the width parameter is $\langle\mathbf{y}^2\rangle = (5/2)\sigma^2$. Using the above-determined width parameter σ, we obtain the following root-mean-square radius for a two-quark $D_{sJ}(2317)$ meson:

$$\begin{aligned}
\langle r_{D_{sJ}}^2\rangle_{\text{two}}^{1/2} &= \frac{1}{\sqrt{2}}\frac{\left(m_c^2+m_s^2\right)^{1/2}}{m_c+m_s}\langle\mathbf{y}^2\rangle^{1/2}\\
&= \frac{\sqrt{5}}{2}\frac{\left(m_c^2+m_s^2\right)^{1/2}}{m_c+m_s}\sigma \approx 0.53\,\text{fm}.
\end{aligned} \quad (11)$$

For a four-quark $D_{sJ}(2317)$ meson, its three size parameters are $\sigma_1 = 1/(\mu_1\omega)^{1/2} \approx 0.42$ fm, $\sigma_2 = 1/(\mu_2\omega)^{1/2} \approx 0.58$ fm, and $\sigma_3 = 1/(\mu_3\omega)^{1/2} \approx 0.6$ fm. The resulting root-mean-square radius of a four-quark $D_{sJ}(2317)$ meson is then

$$\begin{aligned}
\langle r_{D_{sJ}}^2\rangle_{\text{four}}^{1/2} &= \left[\frac{3}{4}\frac{\left(m_c^2+m_s^2\right)\sigma_1^2}{(m_c+m_s)^2}\right.\\
&\quad + \frac{9}{16}\frac{\left((m_c+m_s)^2+2m_q^2\right)\sigma_2^2}{(m_c+m_s+m_q)^2}\\
&\quad \left. + \frac{1}{2}\frac{\left((m_c+m_s+m_q)^2+3m_q^2\right)\sigma_3^2}{(m_c+m_s+2m_q)^2}\right]^{1/2}\\
&\approx 0.62\,\text{fm},
\end{aligned} \quad (12)$$

which is somewhat larger than that of a two-quark $D_{sJ}(2317)$ meson.

The coalescence integral in Eq. (3) can be evaluated analytically if we expand the hyperbolic functions to first order, neglect the transverse flow, and use nonrelativistic momentum distributions for quarks. The first approximation is valid for $|y| \leqslant 0.5$ considered in the present study. Although the transverse flow strongly affects the transverse momentum spectrum of produced $D_{sJ}(2317)$ mesons, it only has a small effect on its number. As shown in Appendix A, these approximations lead to the following numbers of produced $D_{sJ}(2317)$ mesons from quark coalescence: $\sim 1.9 \times 10^{-2}$ for a two-quark $D_{sJ}(2317)$ meson and $\sim 1.1 \times 10^{-3}$ for a four-quark $D_{sJ}(2317)$ meson. These numbers are about a factor of 2 larger than those obtained by numerically evaluating the coalescence integral using the Monte Carlo method of Ref. [15], which gives about 9.8×10^{-3} and 4.2×10^{-4} per unit rapidity for the two-quark and four-quark $D_{sJ}(2317)$ mesons, respectively, largely because of the use of the relativistic quark distribution functions.

Since equilibrium thermal models have been successfully employed in describing the experimental data for the yields and ratios of many hadrons in heavy ion collisions at RHIC [35,36], it is of interest to compare the predicted number of $D_{sJ}(2317)$ mesons from the coalescence model with that from the statistical model. In terms of the charm fugacity γ_C and the strangeness chemical potential μ_S, this model gives the following number of produced $D_{sJ}(2317)$ mesons at hadronization:

$$\begin{aligned} N_{D_{sJ}}^{\rm stat} &= \gamma_C \int_{\sigma_h} \frac{p^\mu d\sigma_\mu}{(2\pi)^3}\frac{d^3\mathbf{p}}{E} f_{D_{sJ}}(x,p) \\ &\approx \frac{V_H \gamma_C e^{\mu_S/T_H}}{(2\pi)^2}\int dm_T m_T^2 \quad e^{-\frac{\bar\gamma_H m_T}{T_H}} I_0\left(\frac{\bar\gamma_H \bar\beta_H p_T}{T_C}\right) \\ &\approx 5.2\times 10^{-2}, \end{aligned} \tag{13}$$

where $f_{D_{sJ}}(x,p)$ is the thermal distribution function of $D_{sJ}(2317)$ mesons, given by an expression similar to Eq. (4) for quarks, and I_0 is the modified Bessel function. In obtaining the numerical value in the last line of above equation, we used $V_H \approx 1908\ \ \mathrm{fm}^3$, $T_H = 175\,\mathrm{MeV}$, $\bar\beta_H = 0.3c$, $\mu_S = 10\,\mathrm{MeV}$, and the charm fugacity $\gamma_C \approx 8.4$. The latter ensures that the numbers of charmed hadrons produced statistically at hadronization is the same as the number of charm quarks N_c in the quark-gluon plasma. Specifically, we have $N_D \approx 1.1$, $N_{D^*} \approx 1.5$, $N_{D_s} \approx 0.31$, and $N_{\Lambda_c} \approx 0.11$, giving a total of about 3 charmed hadrons. We note that the number of $D_{sJ}(2317)$ mesons produced in the statistical model is independent of its quark structure, contrary to that in the coalescence model in which the yield for a two-quark $D_{sJ}(2317)$ meson is about a factor of 20 larger than that for a four-quark one.

IV. HADRONIC EFFECTS ON THE D_{sJ}(2317) MESON

A. D_{sJ}(2317) meson absorption cross sections by hadrons

The abundance of $D_{sJ}(2317)$ mesons can change during the expansion of the hadronic matter as a result of absorption by

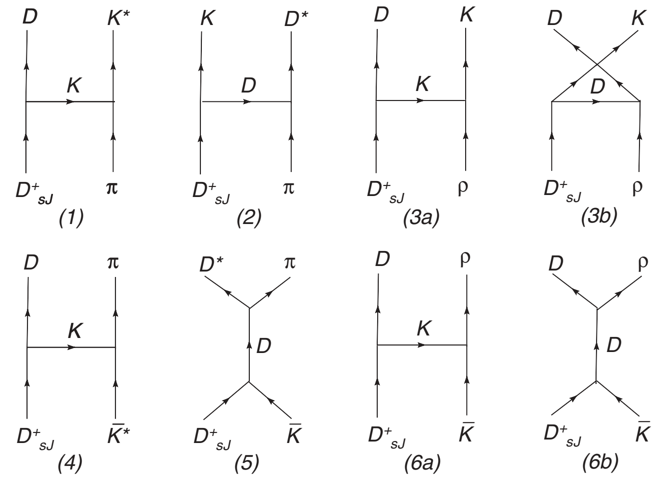

FIG. 2. Born diagrams for $D_{sJ}(2317)$ absorption by π, ρ, $\bar K$, and $\bar K^*$ mesons.

pions, ρ mesons, anti-kaons, and vector anti-kaons. Neglecting reactions with a D_s meson in the final states, which are suppressed as a result of the presence of the isospin violated vertex $D_{sJ} D_s \pi$, we have the following reactions:

$$\begin{aligned} &\pi D_{sJ} \to K D^*(K^* D), \quad \rho D_{sJ} \to K D, \\ &\bar K D_{sJ} \to \rho D(\pi D^*), \quad \bar K^* D_{sJ} \to \pi D, \end{aligned} \tag{14}$$

as shown in Fig. 2 for the lowest-order Born diagrams. The cross sections for these reactions can be evaluated using the interaction Lagrangians

$$\begin{aligned} \mathcal{L}_{\rho KK} &= i g_{\rho KK} (\bar K \vec\tau \partial_\mu K - \partial_\mu \bar K \vec\tau K)\cdot \vec\rho^{\,\mu}, \\ \mathcal{L}_{\rho DD} &= -i g_{\rho DD} (\bar D \vec\tau \partial_\mu D - \partial_\mu \bar D \vec\tau D)\cdot \vec\rho^{\,\mu}, \\ \mathcal{L}_{K^* K\pi} &= i g_{K^* K\pi} \bar K^*_\mu \vec\tau \cdot (K \partial^\mu \vec\pi - \partial^\mu K \vec\pi) + \text{H.c.}, \\ \mathcal{L}_{D^* D\pi} &= -i g_{D^* D\pi} \bar D^*_\mu \vec\tau \cdot (D \partial^\mu \vec\pi - \partial^\mu D \vec\pi) + \text{H.c.}, \\ \mathcal{L}_{D_{sJ} DK} &= g_{D_{sJ} DK} K D D_{sJ}. \end{aligned} \tag{15}$$

In the above, $\vec\tau$ are Pauli matrices for isospin, $\vec\pi$ and $\vec\rho$ denote the pion and ρ meson isospin triplet, respectively, and $K = (K^+, K^0)^T$ and $K^* = (K^{*+}, K^{*0})^T$ denote the pseudoscalar and vector strange meson isospin doublet, respectively. The isospin doublet pseudoscalar D and vector D^* mesons are defined in a similar way. For coupling constants, we use $g_{\rho DD} = 2.52$ from the vector dominance model (VDM) [37,38], $g_{K^*K\pi} = 3.25$ [39] and $g_{D^*D\pi} = 6.3$ [40] from the decay widths of K^* and D^*, respectively, and $g_{\rho KK} = 3.25$ from the SU(3)-flavor symmetry [41]. The coupling constant $g_{KDD_{sJ}}$ has been studied in the QCD sum rules, and its value depends strongly on the quark structure of $D_{sJ}(2317)$ meson. The predicted values are $g_{D_{sJ}DK} = 9.2$ GeV if the $D_{sJ}(2317)$ meson is a two-quark state [42] and $g_{D_{sJ}DK} = 3.15$ GeV if it is a four-quark state [43].

The amplitudes for the reactions shown in Fig. 2 are given by

$$\begin{aligned}
\mathcal{M}_1 &= -\tau^a_{ij} g_{D_{sJ}DK}\, g_{K^*K\pi} \frac{1}{t-m_K^2}(2p_2-p_4)_\mu \epsilon_4^\mu,\\
\mathcal{M}_2 &= \tau^a_{ij} g_{D_{sJ}DK}\, g_{D^*D\pi} \frac{1}{t-m_D^2}(2p_2-p_4)_\mu \epsilon_4^\mu,\\
\mathcal{M}_{3a} &= \tau^a_{ij} g_{D_{sJ}DK}\, g_{\rho KK} \frac{1}{t-m_K^2}(2p_4-p_2)_\mu \epsilon_2^\mu,\\
\mathcal{M}_{3b} &= \tau^a_{ij} g_{D_{sJ}DK}\, g_{\rho DD} \frac{1}{u-m_D^2}(p_2-2p_3)_\mu \epsilon_2^\mu,\\
\mathcal{M}_4 &= \mathcal{M}_1(p_2 \leftrightarrow -p_4),\\
\mathcal{M}_5 &= \mathcal{M}_2(p_2 \leftrightarrow -p_3;\, p_3 \leftrightarrow p_4),\\
\mathcal{M}_6 &= \mathcal{M}_3(p_2 \leftrightarrow -p_4).
\end{aligned} \tag{16}$$

Here, the matrix element τ^a_{ij} takes into account the isospin states of the particles in a reaction, with a denoting those of isospin triplet π and ρ mesons, and i and j those of isospin doublet K, K^*, D, and D^* mesons. The momenta p_1 and p_2 are those of initial (−) state particles, while p_3 and p_4 are those of final (−) state particles on the left and right side of a diagram. The usual Mandelstam variables are given by $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, and $u=(p_1-p_4)^2$.

To obtain the full amplitudes, one needs in principle to carry out a coupled-channel calculation in order to avoid violation of unitarity. Such an approach is, however, beyond the scope of this study. To prevent the artificial growth of the tree-level amplitudes with the energy, we introduce instead form factors at interaction vertices, which are taken to have the form [44]

$$F(\mathbf{q}) = \frac{\Lambda^2}{\Lambda^2+\mathbf{q}^2}, \tag{17}$$

where $\mathbf{q}^2$, taken in the center of mass, is the squared three-momentum transfer for t and u channels, or the squared three-momentum of either the incoming or outgoing particles for the s channel. For the cutoff parameter Λ, we use $\Lambda = 1.3$ GeV for vertices involving an off-shell K meson and $\Lambda = 3.7$ GeV for those involving an off-shell D meson. These values are determined from the QCD sum-rule calculations given in Appendix B for the $D_{sJ}DK$ three-point functions. Although the calculations are only for a two-quark $D_{sJ}(2317)$ meson, we use them also for a four-quark $D_{sJ}(2317)$ meson as well as for other vertices in the diagrams in Fig. 2. We expect this to be a reasonable assumption, because a study of the $X(3872)$ meson in the QCD sum rules has indicated that both the form and the cutoff of its form factor are not significantly different between a two-quark and a four-quark $X(3872)$ meson [45].

The isospin- and spin-averaged cross section is then given by

$$\sigma_n = \frac{1}{64\pi s N_I N_S}\frac{p_f}{p_i}\int d\Omega \overline{|\mathcal{M}_n|^2} F^4, \tag{18}$$

where $\overline{|\mathcal{M}_n|^2}$ denotes the squared amplitude obtained from summing over the isospins and spins of both initial and final particles, with F denoting the appropriate form factors at interaction vertices. The factors $N_I=(2I_1+1)(2I_2+1)$ and $N_S=(2S_1+1)(2S_2+1)$ in the denominator are due to averaging over the isospins I_1 and I_2 as well as the spins S_1 and S_2 of initial particles. The three-momenta in the center of mass of initial and final particles are denoted by p_i and p_f, respectively.

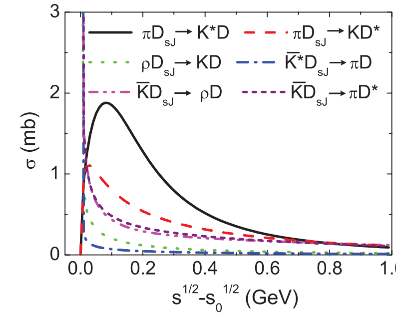

FIG. 3. (Color online) Cross sections for the absorption of a four-quark $D_{sJ}(2317)$ meson by π, ρ, $\bar{K}$, and $\bar{K}^*$ mesons via reaction $\pi D_{sJ} \to KD^*(K^*D)$, $\rho D_{sJ} \to KD$, $\bar{K}D_{sJ} \to \rho D(\pi D^*)$, and $\bar{K}^* D_{sJ} \to \pi D$.

In Fig. 3, we show the absorption cross sections of the $D_{sJ}(2317)$ meson by π, ρ, $\bar{K}$, and $\bar{K}^*$ as functions of the total center-of-mass energy $s^{1/2}$ above the threshold energy $s_0^{1/2}$ of a reaction for the scenario that it is a four-quark state. Aside from those near the threshold of a reaction, where the cross section can be very large or small depending on whether the reaction is exothermic or endothermic, most cross sections are less than 1 mb except the reaction $\pi D_{sJ} \to K^*D$, which has a peak value of about 2 mb. If the $D_{sJ}(2317)$ meson is a two-quark state, its absorption cross sections are about a factor of 9 larger than corresponding ones shown in Fig. 3 as the coupling constant $g_{D_{sJ}DK}$ is about a factor of 3 larger for a two-quark $D_{sJ}(2317)$ meson than for a four-quark one.

The $D_{sJ}(2317)$ meson can also be produced in the hadronic matter by the inverse reactions $KD^*(DK^*) \to \pi D_{sJ}$, $KD \to \rho D_{sJ}$, $\rho D(\pi D^*) \to \bar{K}D_{sJ}$, and $\pi D \to \bar{K}^* D_{sJ}$, with cross sections related to those of absorption reactions via the detailed balance relations.

B. Thermally averaged $D_{sJ}(2317)$ meson absorption cross sections

In the kinetic model to be used in the next section for studying $D_{sJ}(2317)$ meson absorption and production in hadronic matter, the thermally averaged cross sections are needed. In terms of the thermal distribution functions $f_i(\mathbf{p})$ of $D_{sJ}(2317)$ mesons and other hadrons, the thermally averaged cross section $\sigma_{ab\to cd}$ for the reaction $ab \to cd$ is given by [46]

$$\begin{aligned}
\langle \sigma_{ab\to cd} v\rangle &= \frac{\int d^3\mathbf{p}_a d^3\mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b)\sigma_{ab\to cd} v_{ab}}{\int d^3\mathbf{p}_a d^3\mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b)}\\
&= \left[4\alpha_a^2 K_2(\alpha_a)\alpha_b^2 K_2(\alpha_b)\right]^{-1}\\
&\quad \times \int_{z_0}^{\infty} dz [z^2-(\alpha_a+\alpha_b)^2][z^2-(\alpha_a-\alpha_b)^2]\\
&\quad \times K_1(z)\sigma(s=z^2T^2),
\end{aligned} \tag{19}$$

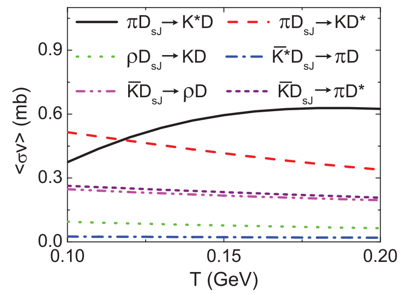

FIG. 4. (Color online) Thermally averaged cross sections for the absorption of a four-quark $D_{sJ}(2317)$ meson by π, ρ, $\bar{K}$, and $\bar{K}^*$ mesons via reaction $\pi D_{sJ} \to KD^*(K^*D)$, $\rho D_{sJ} \to KD$, $\bar{K}D_{sJ} \to \rho D(\pi D^*)$, and $\bar{K}^* D_{sJ} \to \pi D$.

with $\alpha_i = m_i/T$, $z_0 = \max(\alpha_a + \alpha_b, \alpha_c + \alpha_d)$, K_1 being the modified Bessel function, and v_{ab} denoting the relative velocity of the initial two interacting particles a and b, i.e.,

$$v_{ab} = \frac{\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}}{E_a E_b}. \tag{20}$$

The thermally averaged absorption cross sections of the $D_{sJ}(2317)$ meson by π, ρ, $\bar{K}$, and $\bar{K}^*$ as functions of the temperature of the hadronic matter are shown in Fig. 4 for the case that $D_{sJ}(2317)$ is a four-quark meson. It is seen that the thermally averaged cross section of the dominant reaction $\pi D_{sJ} \to K^*D$ has values less than 0.6 mb. Its value is again about a factor of 9 larger if the $D_{sJ}(2317)$ is a two-quark meson.

V. TIME EVOLUTION OF THE $D_{sJ}(2317)$ MESON ABUNDANCE IN HADRONIC MATTER

A. Rate equation for $D_{sJ}(2317)$ meson production in heavy ion collisions

In terms of thermally averaged cross sections and the densities of π, ρ, K, and K^* mesons, the time evolution of the $D_{sJ}(2317)$ meson abundance in the hadronic matter is determined by the kinetic equation

$$\frac{dN_{D_{sJ}}(\tau)}{d\tau} = R_{\rm QGP}(\tau) + \sum_{a,b,c} \langle \sigma_{aD_{sJ}\to bc} v_{aD_{sJ}} \rangle n_a^{(0)}(\tau) \times \left[N_{D_{sJ}}^{(0)}(\tau) \frac{n_c(\tau)}{n_c^{(0)}(\tau)} - N_{D_{sJ}}(\tau) \right], \tag{21}$$

where $n_a^{(0)}(\tau)$, $n_c^{(0)}(\tau)$ and $N_{D_{sJ}}^{(0)}(\tau)$ are, respectively, the equilibrium densities of light meson type a, charmed meson type c, and $D_{sJ}(2317)$ meson in the hadronic matter at proper time τ when its temperature is T according to Eq. (2). These equilibrium densities are calculated using formulas similar to Eq. (13) without the fugacity parameter. Since hadronization of the quark-gluon plasma takes a finite time of $\tau_H - \tau_C \simeq 2.5\,{\rm fm}/c$, $D_{sJ}(2317)$ mesons are produced from the quark-gluon plasma in the mixed phase, with a rate proportional to the volume of the quark-gluon plasma. This is included in

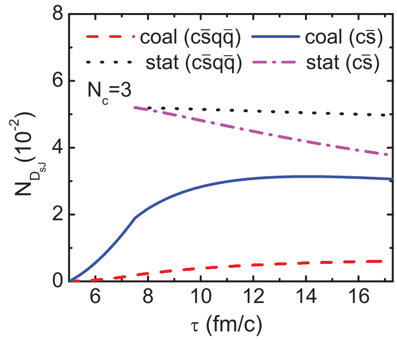

FIG. 5. (Color online) Time evolution of the $D_{sJ}(2317)$ meson abundance in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for different initial numbers of $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma.

Eq. (21) through the term $R_{\rm QGP}(\tau)$. Since the fraction of the quark-gluon plasma during the mixed phase decreases almost linearly with the proper time, we can approximately write

$$R_{\rm QGP}(\tau) = \begin{cases} N_{D_{sJ}}^{(0)}/(\tau_H - \tau_C), & \tau_C < \tau < \tau_H; \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

In the above, $N_{D_{sJ}}^{(0)}$ is the total number of $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma. In the following calculations, it is obtained either from the coalescence model by evaluating Eq. (3) numerically using the Monte Carlo method or from the statistical model using Eq. (13). In writing Eq. (21), we have assumed that the total number of charmed hadrons is conserved during the evolution of the hadronic matter as charms are not likely to be produced and destroyed in the hadronic matter because of their small production and annihilation cross sections [47–49].

B. D_{sJ} meson yield in relativistic heavy ion collisions

In Fig. 5, the abundance of $D_{sJ}(2317)$ mesons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is shown as a function of the proper time of the fire-cylinder. Since the initial number of $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma via quark coalescence is below the equilibrium number for both the two-quark and four-quark $D_{sJ}(2317)$ mesons, its number increases during the hadronic evolution as shown by the solid and dashed lines, respectively. The final number of $D_{sJ}(2317)$ mesons is about 3.0×10^{-2} if the $D_{sJ}(2317)$ meson is a two-quark state and is about 6.0×10^{-3} if it is a four-quark meson. Although the ratio ($\sim$5) between the final numbers for the two- and four-quark $D_{sJ}(2317)$ mesons is smaller than that ($\sim$20) for initially produced $D_{sJ}(2317)$ mesons, it is still appreciable. This result differs significantly from the predictions of the statistical model. In this case, the $D_{sJ}(2317)$ meson number decreases slightly to 3.8×10^{-2} during the hadronic evolution if it is a two-quark meson as shown by the dash-dotted line, and it remains essentially unchanged during hadronic evolution if it is a four-quark meson as shown by the dotted line. Since the final yield of $D_{sJ}(2317)$ mesons in the coalescence model is much smaller for a four-quark state than for a two-quark state and also that from the statistical model, studying $D_{sJ}(2317)$ meson production in relativistic heavy ion collisions thus provides the possibility of understanding not only its production mechanism but also its quark structure.

Above results are obtained by assuming that there are three charm quarks in the quark-gluon plasma, based on the number of charm quarks measured by the PHENIX Collaboration in $p + p$ collisions. If this number is increased by a factor of 2, which is closer to that expected from the STAR experiment on d+Au collisions, the final $D_{sJ}(2317)$ meson numbers in both the coalescence and statistical models are increased by about a similar factor. A similar reduction factor is seen in the final $D_{sJ}(2317)$ meson numbers in heavy ion collisions if the total charm quark number is reduced by a factor of 2 as given by the PYTHIA program for $p + p$ collisions.

VI. SUMMARY

Using the quark coalescence model, we have predicted the yield of $D_{sJ}(2317)$ mesons in central Au+Au collisions at RHIC. Contrary to the prediction of the statistical model, the initial number of $D_{sJ}(2317)$ meson produced at the end of the quark-gluon stage of heavy ion collisions depends sensitively on whether it is a two-quark or a four-quark meson, with the former giving an order of magnitude larger number than the latter. To take into account the effects of absorption and production during subsequent hadronic evolution, we have used a hadronic model to evaluate the cross sections for the absorption of the $D_{sJ}(2317)$ meson by pion, ρ meson, anti-kaon, and vector anti-kaon in the tree-level Born approximation. With empirical masses and coupling constants as well as form factors from the QCD sum rules, we have found that all these cross sections are small, except the reaction $\pi D_{sJ} \to K^* D$, which has a peak cross section of about 2 mb, if the $D_{sJ}(2317)$ is a four-quark meson, but they are about ten times larger if it is a conventional two-quark meson. Including these reactions in a kinetic model based on a schematic hydrodynamic description of relativistic heavy ion collisions, we have studied the time evolution of the abundance of $D_{sJ}(2317)$ mesons in these collisions. Our results show that the large difference in the initial numbers given by the quark coalescence model for the two-quark and four-quark $D_{sJ}(2317)$ mesons remains appreciable at freeze-out. On the other hand, the $D_{sJ}(2317)$ number determined from the statistical model is essentially unchanged if the $D_{sJ}(2317)$ meson is a four-quark meson and only changes slightly if it is a two-quark meson. Studying $D_{sJ}(2317)$ production at RHIC and also at the forthcoming LHC thus offers the possibility to understand its quark structure and production mechanism.

In the present study, we have assumed that quarks in the four-quark $D_{sJ}(2317)$ meson are all in a relative s wave. The possibility that it is formed from a pair of diquark and antidiquark states is not considered. How such a quark structure for the $D_{sJ}(2317)$ meson would affect its production probability is worth studying. Also, to determine the yield of $D_{sJ}(2317)$ mesons in relativistic heavy ion collisions, one requires accurate information on the number of charm quarks produced during initial hard nucleon-nucleon collisions, as increasing or decreasing the initial quark number by a factor affects the final $D_{sJ}(2317)$ meson number by a similar factor. Furthermore, it is important to know if charm quarks can be produced from the quark-gluon plasma, as this would affect the initial $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma as well.

ACKNOWLEDGMENTS

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APPENDIX A: APPROXIMATE EVALUATION OF THE COALESCENCE INTEGRAL

For a hypersurface of constant proper time and a distribution with Bjorken correlation between y and η, we can expand the hyperbolic function in the coalescence integral to first order in y and η if we consider $D_{s,J}(2317)$ meson production at midrapidity with $|y| < 0.5$. In this case, the invariant phase-space factor in Eq. (3) can be approximated by

$$p_i \cdot d\sigma_i \frac{d^3\mathbf{p}_i}{E_i} \simeq d^3\mathbf{x}_i d^3\mathbf{p}_i. \tag{A1}$$

Neglecting the transverse flow and treating quarks nonrelativistically, we can then use the relation

$$\sum_{i=1}^{n} \frac{\mathbf{p}_i^2}{2\,m_i T} = \frac{\mathbf{K}^2}{2M\,T} + \sum_{i=1}^{n-1} \frac{\mathbf{k}_i^2}{2\mu_i T}, \tag{A2}$$

where $M = \sum_{i=1}^{n} m_i$, to express the quark Boltzmann momentum distribution functions in terms of the total $\mathbf{K}$ and relative $\mathbf{k}_i$ momenta. Using also the total and relative $\mathbf{y}_i$ coordinates of the quarks, we obtain the following expression for the number of $D_{s,J}(2317)$ mesons produced from quark coalescence:

$$N_{D_{sJ}} = g_{D_{sJ}} \prod_{j=1}^{n} N_j \prod_{i=1}^{n-1} \frac{\int d^3\mathbf{y}_i\, d^3\mathbf{k}_i\, f_{D_{sJ}}^W(\mathbf{y}_i, \mathbf{k}_i)\, f_q(\mathbf{k}_i)}{\int d^3\mathbf{y}_i d^3\mathbf{k}_i\, f_q(\mathbf{k}_i)}. \tag{A3}$$

In the above, we made use of the fact that the Wigner function of $D_{sJ}(2317)$ meson is factorable in both the relative coordinates and the relative momenta of its constituent quarks. Because of $-0.5 \leqslant y = \eta \leqslant 0.5$, the momentum space integral in Eq. (A3) reduces to a two-dimensional one, as the momentum integral in the z-direction gives simply one.

If the $D_{sJ}(2317)$ meson is a p-wave two-quark meson, evaluating the integrals in Eq. (A3) with its Wigner function given by Eq. (5) leads to the following number of $D_{sJ}(2317)$

mesons produced from quark coalescence:

$$N_{D_{sJ}}^{\rm two} \simeq \frac{1}{36} N_c N_{\bar{s}} \frac{2}{3} \frac{(4\pi\sigma^2)^{3/2} 2\mu T_C \sigma^2}{V_C(1+2\mu T_C \sigma^2)^2} \approx 1.9 \times 10^{-2}. \quad \text{(A4)}$$

For a four-quark $D_{sJ}(2317)$ meson with its Wigner function given by Eq. (6), the number of $D_{sJ}(2317)$ mesons produced from quark coalescence is

$$N_{D_{sJ}}^{\rm four} \simeq \frac{1}{1296} N_c N_{\bar{s}} (N_u N_{\bar{u}} + N_d N_{\bar{d}}) \prod_{i=1}^{3} \frac{\left(4\pi\sigma_i^2\right)^{3/2}}{V_C\left(1+2\mu_i T_C \sigma_i^2\right)} \simeq 1.1 \times 10^{-3}. \quad \text{(A5)}$$

It is interesting to note that the ratio of the yields of the two-quark to the four-quark $D_{sJ}(2317)$ meson is about 17 and is largely due to the different color-spin-isospin statistical factors associated with the two different quark structures of the $D_{sJ}(2317)$ meson.

APPENDIX B: THE $D_{sJ}DK$ FORM FACTOR

In this Appendix, we compute the $D_{sJ}DK$ form factor using the QCD rum rules [50,51]. In this approach, the short-range perturbative QCD is extended by the Wilson's operator product expansion (OPE) of the correlators, which results in a series in powers of the squared momentum with Wilson coefficients. The convergence at low momentum is improved by using a Borel transform. The expansion involves universal quark and gluon condensates. Equating the quark-based calculation of a given correlator to the same correlator that is calculated using hadronic degrees of freedom via a dispersion relation then provides the sum rules from which a hadronic quantity can be estimated.

We shall use the three-point function to evaluate the $D_{sJ}DK$ form factor by following the procedure suggested in Ref. [52] and further extended in Ref. [53]. This means that we shall calculate the correlators for an off-shell D meson and then for an off-shell K meson, requiring that the corresponding extrapolations to the respective poles lead to the same unique coupling constant.

The three-point function associated with a $D_{sJ}DK$ vertex with an off-shell D meson is given by

$$\Gamma_\mu^{(D)}(p,p') = \int d^4x d^4y \langle 0|T\{j_{5\mu}(x) j_D(y) j_{D_{sJ}}^\dagger(0)\}|0\rangle \times e^{ip'\cdot x} e^{i(p-p')\cdot y}, \quad \text{(B1)}$$

where $j_{5\mu} = \bar{s}\gamma_\mu\gamma_5 q$, $j_D = i\bar{q}\gamma_5 c$, and $j_{D_{sJ}} = \bar{c}s$ are the interpolating fields for the K, D, and D_{sJ}, respectively with q, s, and c being the light, strange, and charm quark fields. Here we take D_{sJ} to be a standard scalar quark-antiquark meson.

The phenomenological side of the vertex function, $\Gamma_\mu(p,p')$, is obtained by the consideration of the K and D states' contribution to the matrix element in Eq. (B1):

$$\Gamma_\mu^{(D)\rm phen}(p,p') = \frac{m_{D_{sJ}} m_D^2 F_K f_D f_{D_{sJ}}}{m_c\left(p^2 - m_{D_{sJ}}^2\right)\left(p'^2 - m_K^2\right)} \times \frac{g_{D_{sJ}DK}^{(D)}(q^2)}{\left(q^2 - m_D^2\right)} p'_\mu + \text{higher resonances.} \quad \text{(B2)}$$

In deriving Eq. (B2), we used

$$\langle D_{sJ}(p)|K(p')D(q)\rangle = g_{D_{sJ}DK}^{(D)}(q^2), \quad \text{(B3)}$$

where $q = p' - p$, and the decay constants F_K and f_D and $f_{D_{sJ}}$ are defined by the matrix elements

$$\langle 0|j_{5\mu}|K(p')\rangle = ip'_\mu F_K, \quad \text{(B4)}$$

$$\langle 0|j_D|D(q)\rangle = \frac{m_D^2 f_D}{m_c}, \quad \text{(B5)}$$

and

$$\langle 0|j_{D_{sJ}}|D_{sJ}(p)\rangle = m_{D_{sJ}} f_{D_{sJ}}. \quad \text{(B6)}$$

The contribution of higher resonances and continuum in Eq. (B2) will be taken into account as usual in the standard form of Ref. [54], through the continuum thresholds s_0 and u_0 for the D_{sJ} and K mesons, respectively.

The QCD side, or the theoretical side, of the vertex function is evaluated by performing Wilson's operator product expansion of the operator in Eq. (B1). Expressing Γ_μ in terms of the invariant amplitudes,

$$\Gamma_\mu(p,p') = F_1(p^2, p'^2, q^2) p_\mu + F_2(p^2, p'^2, q^2) p'_\mu, \quad \text{(B7)}$$

we can write a double dispersion relation for each one of the invariant amplitudes F_i over the virtualities p^2 and p'^2 holding $Q^2 = -q^2$ fixed:

$$F_i^{(D)}(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2}\int_{m_c^2}^{\infty} ds \int_0^{\infty} du \frac{\rho_i(s,u,Q^2)}{(s-p^2)(u-p'^2)}, \quad \text{(B8)}$$

where $\rho_i(s,u,Q^2)$ equals the double discontinuity of the amplitude $F_i(p^2, p'^2, Q^2)$ on the cuts $m_c^2 \leqslant s \leqslant \infty$ and $0 \leqslant u \leqslant \infty$, which can be evaluated using Cutkosky's rules. Finally, to suppress the condensates of higher dimension and at the same time reduce the influence of higher resonances, we perform a double Borel transform in both variables $P^2 = -p^2 \to M^2$ and $P'^2 = -p'^2 \to M'^2$. Equating the two representations described above, we obtain the following sum rule in the structure p'_μ:

$$\frac{m_{D_{sJ}} m_D^2}{m_c} F_K f_D f_{D_{sJ}} g^{(D)}_{D_{sJ}DK}(Q^2) e^{-m^2_{D_{sJ}}/M^2} e^{-m_K^2/M'^2} = \left(Q^2+m_D^2\right)\Bigg[m_c\langle\bar{s}s\rangle e^{-m_c^2/M^2} - \frac{1}{4\pi^2}\int_{m_c^2}^{s_0} ds \int_0^{u_{\max}} du \exp(-s/M^2) \times \exp(-u/M'^2) f(s,t,u)\theta(u_0-u)\Bigg], \tag{B9}$$

where $t=-Q^2$ and

$$f(s,t,u) = \frac{3}{2[\lambda(s,u,t)]^{1/2}}\Big(m_c^2+2m_cm_s-s + \left(2m_c^2+2m_cm_s-s-t+u\right)\big(m_c^2(s-t+u)+s(t+u-s)\big)[\lambda(s,u,t)]^{-1}\Big), \tag{B10}$$

with $\lambda(s,u,t)=s^2+u^2+t^2-2su-2st-2tu$, and $u_{\max}=s+t-m_c^2-st/m_c^2$.

We use the same parameters as in Ref. [42]: $m_s = 0.15$ GeV, $m_c = 1.26$ GeV, $F_K = 0.16$ GeV, $m_D = 1.865$ GeV, $m_K = 0.498$ GeV, $m_{D_{sJ}} = 2.317$ GeV, $f_D = 0.23$ GeV, $f_{D_{sJ}} = 0.225$ GeV, and $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, with $\langle\bar{q}q\rangle = -(0.245)^3$ GeV^3. For the continuum thresholds, we take $s_0 = (6.3\pm0.1)$ GeV^2 and $u_0 = (m_K+\Delta u)^2$ with $\Delta u = 0.5$ GeV.

We also use the same Borel window as in Ref. [42], i.e., 10 GeV$^2 \leqslant M^2 \leqslant 20$ GeV^2, and work at a fixed ratio $M'^2/M^2 = 0.64/m^2_{D_{sJ}}$. We find a good Borel stability in this region of the Borel mass. Fixing $M^2=15$ GeV^2, we show by the filled circles in Fig. 6 the momentum dependence of $g^{(D)}_{D_{sJ}DK}(Q^2)$.

Since the present approach cannot be used at small values of Q^2, extracting the $g_{D_{sJ}DK}$ coupling from the form factor requires extrapolation of the curve to the mass of the off-shell meson D. To do this, we fit the QCD sum-rule results with an analytical expression. We obtain a reasonable fit using a monopole form, that is,

$$g^{(D)}_{D_{sJ}DK}(Q^2) = \frac{92.4}{Q^2+14.1}, \tag{B11}$$

where the numbers are in units of GeV^2. This fit is also shown by the solid line in Fig. 6. From Eq. (B11) we get $g_{D_{sJ}DK} = g^{(D)}_{D_{sJ}DK}(Q^2=-m_D^2) = 8.7$.

To check the consistency of this fit, we also evaluate the form factor at the same vertex, but for an off-shell kaon. In this case, we have to evaluate the three-point function

$$\Gamma^{(K)}_\mu(p,p') = \int d^4x d^4y \langle 0|T\{j_D(x) j_{5\mu}(y) j^\dagger_{D_{sJ}}(0)\}|0\rangle \times e^{ip'\cdot x} e^{i(p-p')\cdot y}. \tag{B12}$$

Proceeding in a similar way, we obtain the following sum rule:

$$\frac{m_{D_{sJ}} m_D^2}{m_c} F_K f_D f_{D_{sJ}} g^{(K)}_{D_{sJ}DK}(Q^2) e^{-m^2_{D_{sJ}}/M^2} e^{-m_D^2/M'^2} = -\frac{Q^2+m_K^2}{4\pi^2}\int_{m_c^2}^{s_0} ds \int_{u_{\min}}^{u_0} du e^{-s/M^2} e^{-u/M'^2} \times g(s,t,u), \tag{B13}$$

where $u_{\min} = m_c^2 - \frac{m_c^2 t}{s-m_c^2}$ and

$$g(s,t,u) = \frac{3}{[\lambda(s,u,t)]^{3/2}}[m_c^4(s-t+3u) + u(m_cm_s(s+t-u)+s(-s+t+u)) + m_c^2(-2u(s-t+u)+m_cm_s(s-t+3u))]. \tag{B14}$$

Using now $u_0 = (m_D+\Delta_u)^2$ with $\Delta_u = 0.5$ GeV and $M'^2 = \frac{m_D^2}{m^2_{D_{sJ}}} M^2$, we find that the results are also rather stable as a function of the Borel mass. Fixing $M^2=15$ GeV^2, we show by the squares in Fig. 6 the QCD sum-rule results for $g^{(K)}_{D_{sJ}DK}(Q^2)$. A good fit of these results can be obtained using an exponential form,

$$g^{(K)}_{D_{sJ}DK}(Q^2) = 7.98 e^{-Q^2/1.75}, \tag{B15}$$

where 1.75 is in units of GeV^2, as shown in Fig. 6 by the dashed line. From Eq. (B15) we get $g_{D_{sJ}DK} = g^{(K)}_{D_{sJ}DK}(Q^2=-m_K^2) = 9.1$, in excellent agreement with both the result obtained from $g^{(D)}_{D_{sJ}DK}(Q^2=-m_D^2)$ above and the result obtained in Ref. [42] for this coupling constant: $g_{D_{sJ}DK} = 9.2\pm0.5$.

Considering the uncertainties in the continuum thresholds, and the difference in the values of the coupling extracted when the D meson or the kaon is off-shell, our result for the $D_{sJ}DK$ coupling constant is thus $g_{D_{sJ}DK} = 8.9\pm0.9$.

From the parametrizations in Eqs. (B11) and (B15), we can also obtain information about the cutoff (Λ) in the form factors. We see that the cutoff is much bigger when the D meson is off-shell ($\Lambda \approx 3.7$ GeV) than when the kaon is off-shell ($\Lambda \approx 1.3$ GeV), in agreement with the results obtained in Refs. [52,53].

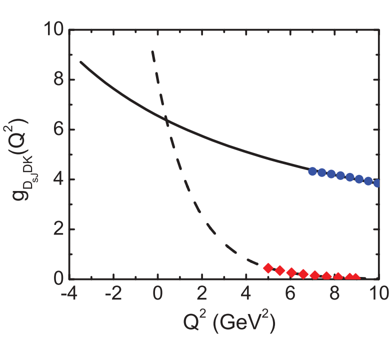

FIG. 6. (Color online) Momentum dependence of the $D_{sJ}DK$ form factors. The solid and dashed lines give the parametrization of the QCDSR results for $g^{(D)}_{D_{sJ}DK}(Q^2)$ (circles) and $g^{(K)}_{D_{sJ}DK}(Q^2)$ (squares), respectively.

[1] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[2] P. Krokovny *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262002 (2003); Y. Mikami *et al.*, *ibid.* **92**, 012002 (2004).
[3] A. Hayashigaki and K. Terasaki, arXiv:hep-ph/0411285 (unpublished).
[4] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); S. Godfrey and R. Kokoski, *ibid.* **43**, 1679 (1991); M. Di Pierro and E. Eichten, *ibid.* **64**, 114004 (2001).
[5] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[6] C. W. Wong and K. F. Liu, Phys. Rev. D **21**, 2039 (1980).
[7] Z. G. Wang and S. L. Wan, Nucl. Phys. **A778**, 22 (2006); M. E. Braco *et al.*, Phys. Lett. **B624**, 217 (2005); H. Kim and Y. Oh, Phys. Rev. D **72**, 074012 (2005).
[8] J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983); **41**, 2236 (1990); E. Swanson, Ann. Phys. (NY) **220**, 73 (1992).
[9] M. Nielsen, Phys. Lett. **B634**, 35 (2006).
[10] W. Wei, P. Z. Huang, and S. L. Zhu, Phys. Rev. D **73**, 034004 (2006).
[11] J. Lu, W. Z. Deng, X. L. Chen, and S. L. Zhu, Phys. Rev. D **73**, 054012 (2006).
[12] G. Rupp and E. van Beveren, Eur. Phys. J. A **31**, 698 (2007).
[13] Z. B. Ma and C. S. Gao, Chin. Phys. Lett. **23**, 568 (2006).
[14] T. S. Biró, P. Lévai, and J. Zimányi, Phys. Lett. **B347**, 6 (1995); Phys. Rev. C **59**, 1574 (1999); J. Zimányi, T. S. Biró, T. Csörgö, and P. Lévai, Phys. Lett. **B472**, 243 (2002); P. Csizmadia and P. Lévai, Phys. Rev. C **61**, 031903(R) (2000).
[15] V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. Lett. **90**, 202302 (2003); Phys. Rev. C **68**, 034904 (2003).
[16] R. C. Hwa and C. B. Yang, Phys. Rev. C **67**, 034902 (2003); **67**, 064902 (2003).
[17] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Phys. Rev. Lett. **90**, 202303 (2003); Phys. Rev. C **68**, 044902 (2003).
[18] L. W. Chen, V. Greco, C. M. Ko, S. H. Lee, and W. Liu, Phys. Lett. **B601**, 34 (2004).
[19] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[20] P. Lévai and U. Heinz, Phys. Rev. C **57**, 1879 (1998).
[21] F. Karsch, Nucl. Phys. **A698**, 199 (2002).
[22] Z. W. Lin, S. Pal, C. M. Ko, B. A. Li, and B. Zhang, Phys. Rev. C **64**, 011902(R) (2001); B. Zhang, C. M. Ko, B. A. Li, and Z. W. Lin, *ibid.* **61**, 067901 (2000); Z. W. Lin, C. M. Ko, B. A. Li, B. Zhang, and S. Pal, *ibid.* **72**, 064901 (2005).
[23] L. Grandchamp and R. Rapp, Phys. Lett. **B523**, 60 (2001); R. Rapp, Phys. Rev. C **63**, 054907 (2001).
[24] L. W. Chen and C. M. Ko, Phys. Rev. C **73**, 044903 (2006).
[25] V. Greco, C. M. Ko, and R. Rapp, Phys. Lett. **B595**, 202 (2004).
[26] F. Cooper and G. Frye, Phys. Rev. D **10**, 186 (1974).
[27] S. S. Adler (PHENIX Collaboration), Phys. Rev. C **72**, 024901 (2005).
[28] F. Laue (STAR Collaboration), J. Phys. G: Nucl. Part. Phys. **31**, S27 (2005).
[29] B. Zhang, L. W. Chen, and C. M. Ko, Phys. Rev. C **72**, 024906 (2005); Nucl. Phys. **A774**, 665 (2006).
[30] D. Molnar, Nucl. Phys. **A774**, 257 (2006).
[31] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **97**, 252002 (2006).
[32] J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **94**, 062301 (2005).
[33] C. Dover, U. Heinz, E. Schnedermann, and J. Zimányi, Phys. Rev. C **44**, 1636 (1991).
[34] C. W. Hwang, Eur. Phys. J. C **23**, 585 (2002).
[35] P. Braun-Munzinger, D. Magestro, K. Redlich, and J. Stachel, Phys. Lett. **B518**, 41 (2001).
[36] G. Torrieri and J. Rafelski, J. Phys. G: Nucl. Part. Phys. **30**, 557 (2004).
[37] Z. Lin and C. M. Ko, Phys. Rev. C **62**, 034903 (2000).
[38] Z. W. Lin, C. M. Ko, and B. Zhang, Phys. Rev. C **61**, 024904 (2000).
[39] C. M. Ko and D. Seibert, Phys. Rev. C **49**, 2198 (1994).
[40] S. Ahmed *et al.* (CLEO Collaboration), Phys. Rev. Lett. **87**, 251801 (2001).
[41] C. H. Li and C. M. Ko, Nucl. Phys. **A712**, 110 (2002).
[42] Z. G. Wang and S. L. Wan, Phys. Rev. D **73**, 094020 (2006).
[43] M. Nielsen, arXiv:hep-ph/0610320, in *Proceedings of the 36th International Symposium on Multiparticle Dynamics, ISMD 2006*, 9–11 March 2006, Kochi, Japan (unpublished).
[44] W. Liu, C. M. Ko, and S. H. Lee, Nucl. Phys. **A724**, 375 (2003); **A728**, 457 (2003).
[45] F. S. Navarra and M. Nielsen, Phys. Lett. **B639**, 272 (2006).
[46] C. M. Ko, X. N. Wang, B. Zhang, and X. F. Zhang, Phys. Lett. **B444**, 237 (1998).
[47] Z. W. Lin and C. M. Ko, Phys. Rev. C **62**, 034903 (2000).
[48] W. Liu, C. M. Ko, and Z. W. Lin, Phys. Rev. C **65**, 015203 (2001).
[49] W. Liu and C. M. Ko, Phys. Lett. **B533**, 259 (2002).
[50] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[51] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[52] M. E. Bracco, M. Chiapparini, A. Lozea, F. S. Navarra, and M. Nielsen, Phys. Lett. **B521**, 1 (2001).
[53] F. S. Navarra, M. Nielsen, M. E. Bracco, M. Chiapparini, and C. L. Schat, Phys. Lett. **B489**, 319 (2000); F. S. Navarra, M. Nielsen, and M. E. Bracco, Phys. Rev. D **65**, 037502 (2002); R. D. Matheus, F. S. Navarra, M. Nielsen, and R. Rodrigues da Silva, Phys. Lett. **B541**, 265 (2002).
[54] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).